Describe the effects on output per worker (y) and capital per worker (k) by changes in savings rate (s), labor growth rate (n), depreciation rate (δ), and technical progress (A). There should be eight multipliers. Determine if each is either positive or negative and show the results in a diagram. For the first three exogenous variables (s, n, δ), use a general function $f(k)$ with constant returns to scale. For technical progress (A), use the Cobb-Douglass production function $y = f(k) = Ak^{\alpha}$.

Start with a general production function showing output (Y) as a function of capital (K) and labor (N):

$$Y = F(K, N) \tag{1}$$

Note that output per worker (y) is equal to output divided by the number of workers:

$$y = \frac{Y}{N} = \frac{F(K,N)}{N} \tag{2}$$

Since the production function has constant returns to scale, we can rewrite this as:

$$y = \frac{F(K,N)}{N} = F\left(\frac{K}{N}, \frac{N}{N}\right) \tag{3}$$

Note that the second argument to the production function is now 1 and the first argument is simply the capital per worker (k). Therefore, equation (3) can be rewritten as a new function of only one variable:

$$y = f(k) \tag{4}$$

This function is assumed to have diminishing returns:

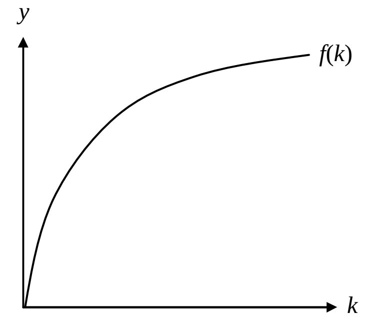


Figure 1. Diminishing Returns

Before going on, realize that there are three other "inputs" to this model, but all are treated as constant (i.e., not explicitly part of the new function). These include the savings rate (s), the growth rate of labor (n), and the depreciation rate (δ). These factor into the model when we look at how K and N change over time:

$$\frac{dK}{dt} = I - \delta K \tag{5}$$

$$\frac{dN}{dt} = nN \tag{6}$$

These formulas should be intuitive. Capital (K) increases by the amount of new investment (I) and decreases by the amount of depreciation, which is simply the depreciation rate times the current amount of capital (i.e., δK). The labor force grows by the labor growth rate times the current number of workers (i.e., nN). Notice that equation (5) can be rewritten using the savings rate (s) and the fact that $I = sY$:

$$\frac{dK}{dt} = sY - \delta K \tag{7}$$

What we really want to look at, however, is how capital per worker (k) changes over time. Note that k is a function of both K and N, so we must take the total derivative (the sum of all the partial derivatives; Simon and Blume p.311), made slightly more difficult by also having to apply the chain rule:

$$\frac{dk}{dt} = \frac{dk}{dK} \cdot \frac{dK}{dt} + \frac{dk}{dN} \cdot \frac{dN}{dt} = \frac{1}{N}\left(sY - \delta K\right) + \frac{-K}{N^2}\left(nN\right) \tag{8}$$

This equation simplifies to:

$$\frac{dk}{dt} = s\frac{Y}{N} - \left(n + \delta\right)\frac{K}{N} \tag{9}$$

Which can further be simplified by using the output per worker ($y = f(k)$) and capital per worker (k):

$$\frac{dk}{dt} = sf\left(k\right) - \left(n + \delta\right)k \tag{10}$$

Now to determine how savings, labor growth, and depreciation rates (s, n, and δ, respectively) affect y and k, we need to consider the steady state scenario. That is, the point where k is not changing over time:

$$\frac{dk}{dt} = sf(k) - (n+\delta)k = 0 \tag{11}$$

Note that at this point, the increase in capital per worker (*sf*(*k*), also called actual investment) is equal to the decrease in capital per worker ((*n* + *δ*)*k*, also called break-even investment). This decrease is made up of depreciation and capital transferred to new workers. The point where they are equal (i.e., actual investment equals break-even investment) is denoted by *k** and is called the balanced growth path (Romer p.17). Graphically:

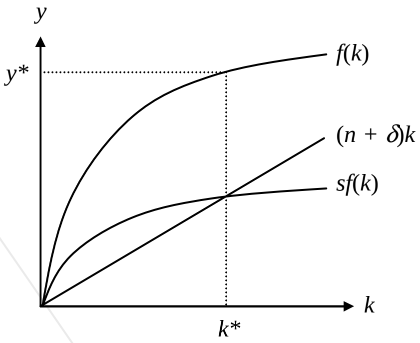


Figure 2. Break Even Investment

To look at the change in *s*, *n*, and *δ*, we have to take the total derivative of equation (11). (Recall this means we sum all the partial derivatives.)

$$\overbrace{sf'(k)dk-(n+\delta)dk}^{\frac{\partial(dk/dt)}{\partial k}} \quad \overbrace{+\ f(k)ds}^{\frac{\partial(dk/dt)}{\partial s}} \quad \overbrace{-\,kdn}^{\frac{\partial(dk/dt)}{\partial n}} \quad \overbrace{-\,kd\delta}^{\frac{\partial(dk/dt)}{\partial \delta}} = 0 \tag{12}$$

Now rearrange terms to find *dk*. The terms that are multiplied by *ds*, *dn*, and *dδ* are the multipliers (i.e., they show how *k* changes when *s*, *n*, and *δ* change, respectively).

$$[(n+\delta) - sf'(k)]dk = f(k)ds - kdn - kd\delta \tag{13}$$

$$dk = \left[\frac{f(k)}{n+\delta-sf'(k)}\right]ds + \left[\frac{-k}{n+\delta-sf'(k)}\right]dn + \left[\frac{-k}{n+\delta-sf'(k)}\right]d\delta \tag{14}$$

From here we can find how these inputs effect output per worker (*y*) by taking the derivative of equation (4):

$$dy = f'(k)dk \tag{15}$$

Plugging equation (14) into equation (15) results in:

$$dy = \left[\frac{f(k)f'(k)}{n+\delta - sf'(k)} \right] ds + \left[\frac{-kf'(k)}{n+\delta - sf'(k)} \right] dn + \left[\frac{-kf'(k)}{n+\delta - sf'(k)} \right] d\delta \qquad (16)$$

Let's look at how each factor affected k and y:

Savings Rate (s)

k Multiplier: $\left[\frac{f(k)}{n+\delta - sf'(k)} \right] > 0$ y Multiplier: $\left[\frac{f(k)f'(k)}{n+\delta - sf'(k)} \right] > 0$

In order to determine the signs of these multipliers mathematically, we need to determine whether the numerators and denominators are positive or negative. Since both share the same denominator, we should start there. Note that the first part ($n + \delta$) is simply the slope of the break-even investment curve (i.e., the straight line in Figure 2). The second part ($sf'(k)$) is the slope of the actual investment curve (i.e., the shallower curve in Figure 2). In the area around the balanced growth path (i.e., k^*), this difference is positive because the straight line is increasing faster than the curved one. Therefore, both multipliers have positive denominators.

It should be fairly obvious that both multipliers also have positive denominators since the production function, $f(x)$, is never negative and the slope, although diminishing, is always positive. With positive numerators and positive denominators, both multipliers are positive. This makes sense, because increased savings rate should result in more capital per worker, hence more output per worker. The effects on k and y can be shown graphically as well:

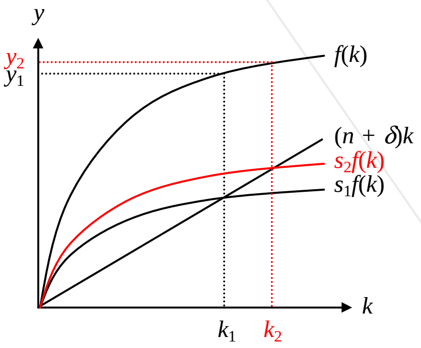


Figure 3. Increased Savings Rate

Labor Growth Rate (n)

k Multiplier: $\left[\frac{-k}{n+\delta - sf'(k)} \right] < 0$ y Multiplier: $\left[\frac{-kf'(k)}{n+\delta - sf'(k)} \right] < 0$

Here again we want to look at the numerators and denominators separately first. Both multipliers share the same denominator which is positive as discussed in the previous section. Both numerators are negative. This is clear for the k Multiplier since k is positive and it has a negative sign in the multiplier. For the y Multiplier, -k is multiplied by $f'(k)$ which is positive and the resulting product is negative.

Intuitively, this result makes sense because if the labor growth rate increases (all else being equal), there will be less capital per worker (i.e., $k\downarrow$) and less output per worker (i.e., $y\downarrow$). The effects on k and y can be shown graphically as well:

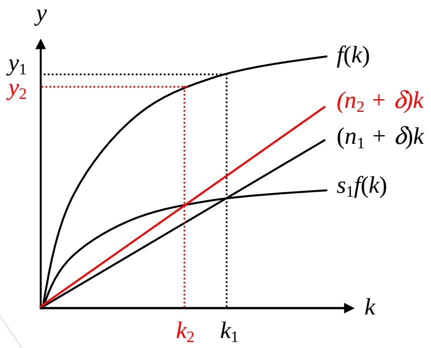


Figure 4. Increased Labor Growth Rate

Depreciation Rate (δ)

k Multiplier: $\left[\dfrac{-k}{n+\delta-sf'(k)}\right]<0$

y Multiplier: $\left[\dfrac{-kf'(k)}{n+\delta-sf'(k)}\right]<0$

An increase in the depreciation rate has the exact same effect as an increase in the labor growth rate. Even the graphs look the same, except for the fact that δ is changing instead of n:

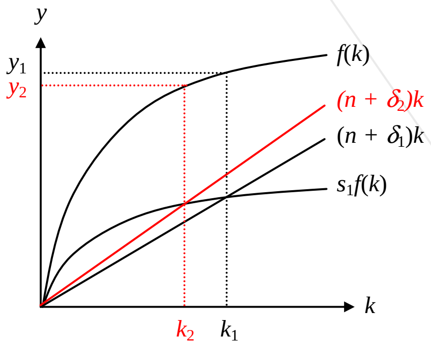


Figure 5. Increased Depreciation Rate

Technical Progress (A)

We now turn our attention to the impact of technical progress (A), by looking at a Cobb-Douglass production function $y = f(k) = Ak^{\alpha}$. This function is substituted into equation (11) and we follow along similar reasoning we used before to find dk in terms of dA:

$$\frac{dk}{dt} = sAk^{\alpha} - (n+\delta)k = 0 \tag{17}$$

Although it is not necessary at this point to also look at the change in s, n, and δ, we might as well in order to double check the generic multipliers generated earlier. Taking the total derivative of equation (17) yields:

$$\overbrace{sA\alpha k^{\alpha-1}dk - (n+\delta)dk}^{\frac{\partial(dk/dt)}{\partial k}} \quad \overbrace{+\,Ak^{\alpha}ds}^{\frac{\partial(dk/dt)}{\partial s}} \quad \overbrace{-\,kdn}^{\frac{\partial(dk/dt)}{\partial n}} \quad \overbrace{-\,kd\delta}^{\frac{\partial(dk/dt)}{\partial \delta}} \quad \overbrace{+\,sk^{\alpha}dA}^{\frac{\partial(dk/dt)}{\partial A}} = 0 \tag{18}$$

Now rearrange terms to find dk. The terms that are multiplied by ds, dn, $d\delta$, and dA are the multipliers.

$$\left[(n+\delta) - sA\alpha k^{\alpha-1}\right]dk = Ak^{\alpha}ds - kdn - kd\delta + sk^{\alpha}dA \tag{19}$$

$$dk = \left[\frac{Ak^{\alpha}}{n+\delta-sA\alpha k^{\alpha-1}}\right]ds + \left[\frac{-k}{n+\delta-sA\alpha k^{\alpha-1}}\right]dn + \left[\frac{-k}{n+\delta-sA\alpha k^{\alpha-1}}\right]d\delta + \left[\frac{sk^{\alpha}}{n+\delta-sA\alpha k^{\alpha-1}}\right]dA \tag{20}$$

Note that the multipliers for s, n, and δ are the same as the ones derived earlier when you substitute $f(k) = Ak^{\alpha}$ and $f'(k) = A\alpha k^{\alpha-1}$.

There is now an additional consideration before moving on to find dy. We have to treat y as a function of both A and k so we must once again take a total derivative:

$$dy = f_k'(k)dk + f_A'(k)dA = A\alpha k^{\alpha-1}dk + k^{\alpha}dA \tag{21}$$

Plugging equation (20) into equation (21) results in:

$$dy = \left[\frac{A^2\alpha k^{2\alpha-1}}{n+\delta-sA\alpha k^{\alpha-1}}\right]ds + \left[\frac{-A\alpha k^{\alpha}}{n+\delta-sA\alpha k^{\alpha-1}}\right]dn + \left[\frac{-A\alpha k^{\alpha}}{n+\delta-sA\alpha k^{\alpha-1}}\right]d\delta + \left[\frac{sA\alpha k^{2\alpha-1}}{n+\delta-sA\alpha k^{\alpha-1}} + k^{\alpha}\right]dA \tag{22}$$

Summarizing the impact of A:

k Multiplier: $\left[\frac{sk^{\alpha}}{n+\delta-sA\alpha k^{\alpha-1}}\right]>0$

y Multiplier: $\left[\frac{sA\alpha k^{2\alpha-1}}{n+\delta-sA\alpha k^{\alpha-1}}+k^{\alpha}\right]>0$

$$=\left[\frac{(n+\delta)k^{\alpha}}{n+\delta-sA\alpha k^{\alpha-1}}\right]$$

Following the same logic used when looking at the multipliers for s, the denominators of both multipliers here are positive. The numerators are also positive since each individual term is positive. The multiplier for y adds the term k^{α} which is also positive. Intuitively, this result makes sense because if there is technological progress, we would expect the amount of capital per worker and the output per worker to increase. Graphically this result is slightly different than the others because the change in A directly effects $f(k)$ so there are two shifts:

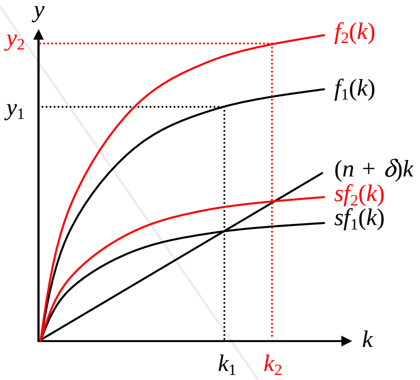


Figure 6. Increased Technological Progress

Documentation

This was compiled predominantly from class notes. Where noted the course text (*Advanced Macroeconomics* by Romer) and *Mathematics for Economists* (Carl P. Simon and Lawrence Blume) were used.

Specific help was received from Professor Bomberger in class. He cleared up where we were starting from to get $dk/dt = 0$. He also clarified some of the points on technological progress.

Guille Sabbioni and I discussed figuring out why the denominators were positive. I knew from class that they were, but didn't recall why. Guille also caught my error in not taking the derivative of y with respect to both k and A in the last section. I double checked that with Professor Bomberger.

Conclusions from Solow Model

Exogenous - "outside the model"; inputs to the model; values taken as given and not determined by the model; for Solow: ds & dn

Endogenous - "inside the model"; outputs of the model; values determined by the model; for Solow: dy & dk

Steady State - eventually growth in output, growth in capital, and growth in labor are the same ($\Delta Y/Y = \Delta K/K = \Delta N/N = n$)

Increase in savings rate (s ↑) - growth of output and capital increases for a while, but eventually levels off at original rate (n); but now have a higher y and k than would've had before (one time increase in standard of living)
NOTE: increase in y is modest (e.g., 50% increase in s yields 14% increase in y) and is slow to materialize

Increase in labor growth rate (n ↑) - results in faster growth of output and capital (equal to the new n), but lower output per worker (Y/N ↓) and lower standard of living

Technological Progress (A) - A is index of technological progress: $A_T = (1+g)A_{T-1}$

Several ways to incorporate it:

$Y = F(K,N)$ - example: $AK^{\alpha}N^{1-\alpha}$

$Y = F(K,AN)$ - labor-augmenting technical change; preferred for convenience although mathematically, all are equivalent for Cobb-Douglas function

$Y = F(AK,N)$ - capital-augmenting technical change

Now $y = \frac{Y}{AN}$ and $k = \frac{K}{AN}$, where AN = effective labor

$$\frac{dk}{dt} = sf(k) - (n + \delta + g)k$$

Difference From No A in Model - $\frac{\Delta Y}{Y} = n + g$ (used to equal n)

IS-LM Model

Solow Assumptions - demand irrelevant in long run; assumes economy is operating at potential GDP; concerned with growth

IS-LM Assumptions - supply is irrelevant in short run; assumes economy is operating below potential (i.e., have excess capacity to absorb any increase in demand); concerned with fluctuations in business cycle (based solely on aggregate demand)

Consumption Function - relationship between consumption & those economic variables that determine decision to consume; we only consider it a function of **disposable income**:

$C = C(Y - T(Y))$ (poor notation; using C as consumption and as function for consumption)

Marginal Propensity to Consume (MPC; C') - amount of increased consumption that results from an increase in income; derivative of consumption function with respect to income; assume **$0 < C' < 1$**

Marginal Propensity to Save (MPS) - $MPC + MPS = 1$; i.e., $MPS = 1 - C'$

Investment - has multiple meanings, but for economists, it means using productive capacity to build capital goods (vs. consumption goods); for now treat as exogenous (given)

Planned Investment - amount businesses want to spend on capital goods, including amount they want to add to their inventories; decide to buy capital goods because they foresee profits accruing to them from using these capital goods

Unplanned Investment - amount businesses have to add or take away from their inventories to make up for excess supply or demand

Equilibrium - in goods market occurs when unplanned investment doesn't exist

Simple Model - no taxes, no government purchases, closed economy

2 equations, 2 unknowns (C and Y): **$C = C(Y)$** and **$Y = C + I$**

Take derivatives: $dC = C'dY$ and $dY = dC + dI$

Sub dC into dY equation and solve for investment multiplier:

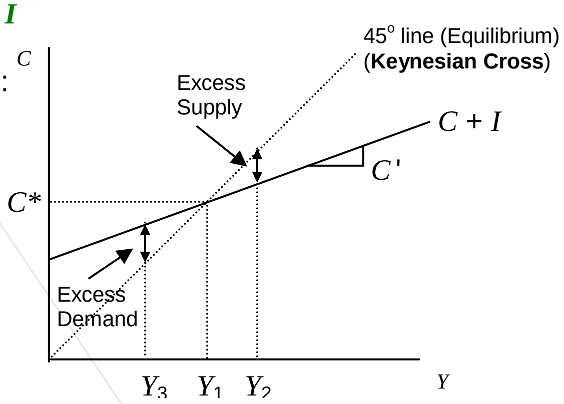
$$dY = C'dY + dI \quad \Rightarrow \quad \frac{dY}{dI} = \boxed{\frac{1}{1-C'} > 0}$$

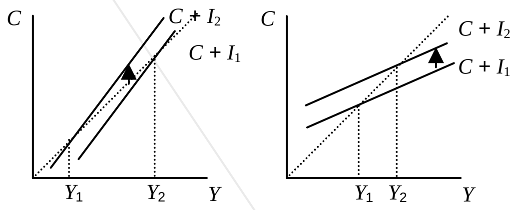
Sign - $1 - C'$ is > 0 because $0 < C' < 1$ (assumption)

Graph - plots consumption (demand, $C + I$) on vertical axis and production (supply, Y) on horizontal axis; for equilibrium (supply = demand) must be on 45° line; Y_2 has excess supply; Y_3 has excess demand

Change in Investment on Graph - $I\uparrow$ shifts curve up and increase equilibrium Y

Multiplier - $I\uparrow \Rightarrow Y\uparrow$ by inverse of MPS... the more people consume (i.e., steeper $C + I$), the larger impact an increase in I will have on output (see smaller graphs)





Adding Fiscal Policy - **$Y = C + I + G$** and **$C = C(Y - T)$**

T = net taxes (tax revenues minus transfer payments)

G = government purchases

Disposable Income - $Y - T$

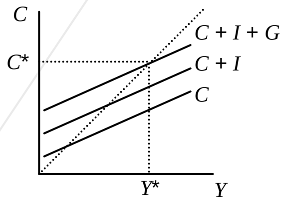
Still 2 equations, 2 unknowns

Take derivatives: $dC = C'dY - C'dT$ and $dY = dC + dI + dG$

Sub dC into dY equation and solve for investment multiplier:

Will get more complicated in a bit

$$dY = C'dY - C'dT + dI + dG \quad \Rightarrow \quad dY = \boxed{\begin{array}{c}\frac{1}{1-C'}\\ >0\end{array}}dI + \boxed{\begin{array}{c}\frac{1}{1-C'}\\ >0\end{array}}dG + \boxed{\begin{array}{c}\frac{-C'}{1-C'}\\ <0\end{array}}dT$$



Tax Cut or Government Purchases? looking at multipliers, G has larger impact on Y (note that dT is multiplied by $C' < 1$); that means, dollar for dollar, government purchases are more effective than tax cuts because with cuts, people retain some of the money (determined by MPS) whereas money from G goes straight into Y

Balanced Budget? if you have increased taxes to cover increased government purchases (i.e., $dG = dT$), there is no change in Y... $1/(1 - C') - C'/(1 - C') = (1 - C')/(1 - C') = 1$

Purpose - output and employment are very sensitive to changes in investment which is volatile; changes in G and T can be used to stabilize output and employment

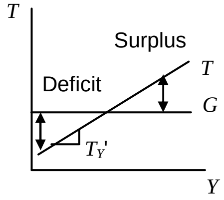
Taxes as Function of Income - $\mathbf{Y = C + I + G}$ and $\mathbf{C = C(Y - T(Y))}$

Realistic - Congress sets tax rates and policies for transfer payments, but actually amount collected and paid depends on Y

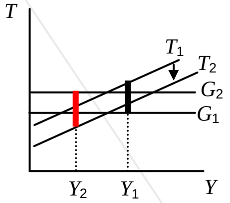
Automatic Stabilizer - net taxes rise as Y↑ because the government collects more tax dollars and makes fewer transfer payments; if Y↓, taxes collected automatically go down and transfer payments go up; not good for government's fiscal position, but credited with minimizing fluctuations in business cycle (difference between peak and trough); want fiscal policy to be automatic because political system is too slow to build consensus (e.g., deciding which taxes to cut)

Surplus - $T - G > 0$; i.e., transfer payments plus government purchases are less than taxes: $G + Tr < Tx$; or look at net taxes: $G - T < 0$

Deficit - $T - G < 0$; i.e., transfer payments plus government purchases are more than taxes: $G + Tr > Tx$; or look at net taxes: $G - T > 0$



2002 Deficit - G↑, T↓, and Y↓ at same time



Marginal Tax Rate (T') - assume $\mathbf{0 < T' < 1}$; marginal rate is higher than you think because it incorporates transfer payments; also called marginal propensity to tax (MPT)

Still 2 equations, 2 unknowns:

Take derivatives: $dC = C'(1 - T')dY - C'dT$ and $dY = dC + dI + dG$

Sub dC into dY equation and solve for investment multiplier:

Poor notation (again)
This is 1 minus the product of C' and $(1 - T')$, <u>not</u> C' evaluated at $(1 - T')$

$$dY = C'(1 - T')dY - C'dT + dI + dG \quad \Rightarrow$$

$$dY = \boxed{\begin{array}{c}\frac{1}{1-C'(1-T')}\\ >0\end{array}}dI + \boxed{\begin{array}{c}\frac{1}{1-C'(1-T')}\\ >0\end{array}}dG + \boxed{\begin{array}{c}\frac{-C'}{1-C'(1-T')}\\ <0\end{array}}dT$$

Impact of Taxes - all multipliers are smaller now because people have less disposable income (effectively reduces MPC)

Adding Money - can't talk about fiscal policy without looking at money; will now look at investment as endogenous (explained by model)

Investment Function - relationship between investment demand (I) & those economic variables that determine the decision by firms to purchase capital goods; $\boldsymbol{I = I(i - \pi_e)}$ (poor notation; using I as investment and as function for investment)

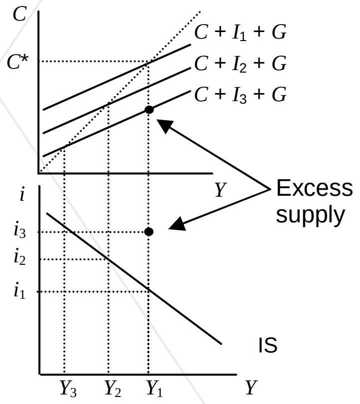
Real Interest Rate (*r*) - difference between interest rate (i) and inflation rate ($\pi = (dP/dt)/P$); for model, use expected inflation (π_e) because decisions made before inflation is known

Investment and Interest - assume increased interest rates reduce investment (i.e., $i\uparrow \Rightarrow Y\downarrow$ or $I_i < 0$)

Back to Model - now have $\boldsymbol{Y = C(Y - T(Y)) + I(i - \pi_e) + G}$; Y and i are endogenous so we have 1 equation and 2 unknowns; need to look at all goods-market equilibria (IS curve)

IS Curve - combinations of interest rate (i) & income (Y) that generate goods-market equilibrium (i.e., [1] aggregate supply = aggregate demand; [2] $Y = C + I + G$; [3] planned investment = savings; [4] unplanned investment = 0); downward sloping because high interest rates discourage investment & therefore reduce equilibrium income; slope of IS curve shows how much equilibrium income will change with change in interest rate; gets name from planned investment (I_p) equals saving (S)

Saving - part of income that is not used for consumption; $\boldsymbol{S = Y - C}$; condition for goods-market equilibrium is saving equals planned investment ($S = I_p$)



Can derive IS curve by using aggregate expenditures (Keynsian cross) curve above the IS curve. Line up income (Y) for various values of interest rate (i).

Shifts in IS - IS↑ (i.e., curve shifts to the right) if $T\downarrow$ ($C\uparrow$), or $\pi_e\uparrow$ ($I\uparrow$), or $G\uparrow$; results in larger output (Y) for given interest rate (i)

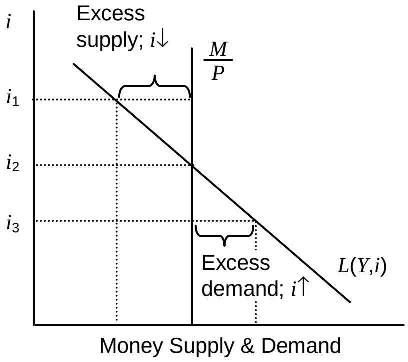
Money-Market Equilibrium - IS curve doesn't give a specific equilibrium, but a set of possible goods-market equilibria; to find a specific equilibrium point, you need to find the equilibrium in the money-market

Money Market - where people increase or decrease the amount of money they hold by selling or buying short-term bonds (e.g., T-bills)

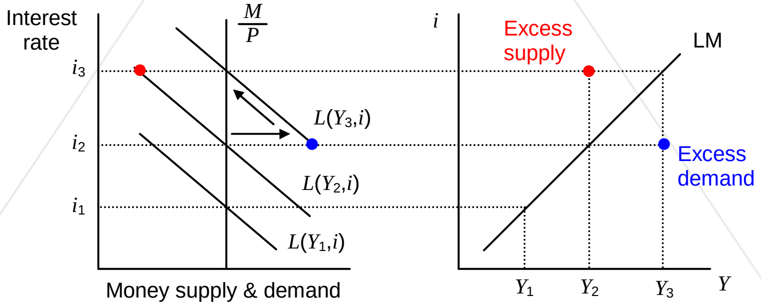
Money - has multiple meanings: wealth (stock), income (flow), etc., but for economists, it means liquid portion of wealth (cash, checking balances, etc.)

M1 - purely transaction-based definition; currency plus demand deposits & travelers checks

    - **M2** - purely transaction-based (M1) plus easily transferable savings accounts (e.g., overnight repurchase agreements, US dollar accounts in Europe, money-market mutual funds, savings deposits, small time deposits)
    - **M3** - everything in M2 plus large time deposits & other accounts used less frequently for transactions purposes
    - **Credit Cards** - affect how much money people want to hold, but are excluded from definition of money because they're not assets
    - **Benefit of Money** - certainty that asset can be quickly & readily used to purchase goods & services
    - **Cost of Money** - holding money costs because it earns no interest or has very low interest rate
  - **Demand for Money (L)** - portion of our wealth we want to hold in the form of money; use L for "liquidity preference"; function of income (# of transactions) and interest rate (cost of holding money): $\mathbf{L = L(Y, i)}$ Note: $i\uparrow \Rightarrow L\downarrow$ and $Y\uparrow \Rightarrow L\uparrow$, so $L_i < 0$ and $L_Y > 0$
  - **Supply of Money (M)** - determined by central bank (Fed); treat as fixed in short run
  - **Constant Purchasing Power** - look at supply of money based on purchasing power by dividing by price level, P
  - **Money-Market Equilibrium** - if there is less or more money demanded than available, actions of money holders in trying to acquire or get rid of money will bring about a change in interest rate & hence quantity of money demanded; if excess demand for money then interest rate is too low (everyone wants more money than is available; acquire money by selling bonds which drives up interest rate on bonds)
    - **Theater Analogy** - only limited number of seats (fixed supply); demand can't create extra seats, so ticket prices are bid up by those who want to attend until demand is brought into line with supply



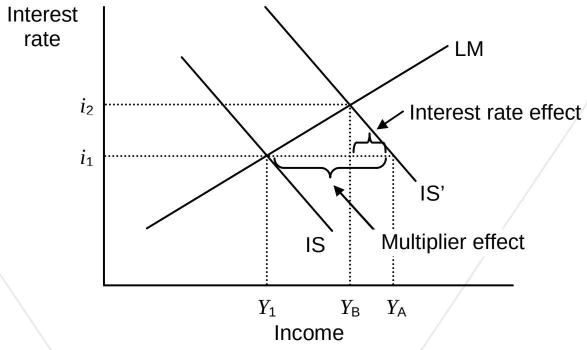
- **LM Curve** - combinations of interest rate (i) & income (Y) that generate money-market equilibrium (i.e., [1] supply of money = demand for money; [2] $\mathbf{L(Y, i) = M/P}$); upward-sloping because higher income ($Y\uparrow$) causes higher demand for money ($L\uparrow$) which causes higher interest rate ($i\uparrow$) to bring money demand back down to equilibrium with fixed supply; name comes from M for money supply & L for money demand



  - **Shifts in LM** - LM↑ (i.e., curve shifts to the right) if $M\uparrow$, $P\downarrow$, or L↓; results in larger output (Y) for given interest rate (i)

IS-LM Framework - point of intersection of IS & LM schedules is one combination of interest rate & income common to both schedules ⇒ point where both goods market & money market are in equilibrium

- **Multiplier Effect** - change in income that would occur following a shift in goods market if there were no change in the interest rate (i.e., ignore asset market)
- **Interest-Rate Effect** - following shift in conditions in goods market, interest-rate effect is change in income resulting from change in interest rate



Back to Model - now have 2 equations, 2 unknowns:

$Y = C(Y - T(Y)) + I(i - \pi_e) + G$ and $L(Y, i) = M/P$

Take derivatives: $dY = C'(1 - T')dY + I'di - I'd\pi_e + dG$ and $L_YdY + L_idi = dM/P - (M/P^2)dP$

Note: $I' = dI/dr$, $r = i - \pi_e$

Solve for dY

$$dY = \frac{I'}{1-C'(1-T')}di - \frac{I'}{1-C'(1-T')}d\pi_e + \frac{1}{1-C'(1-T')}dG$$

Then sub in di equation

$$\frac{I'L_Y}{1-C'(1-T')}di - \frac{I'L_Y}{1-C'(1-T')}d\pi_e + \frac{L_Y}{1-C'(1-T')}dG + L_idi = dM/P - (M/P^2)dP$$

Solve for di

$$\frac{L_i[1-C'(1-T')]+I'L_Y}{1-C'(1-T')}di = \frac{1}{P}dM - \frac{M}{P^2}dP + \frac{I'L_Y}{1-C'(1-T')}d\pi_e + \frac{-L_Y}{1-C'(1-T')}dG$$

$$di = \boxed{\underset{<0}{\frac{1-C'(1-T')}{[L_i(1-C'(1-T'))+I'L_Y]P}}}dM + \boxed{\underset{>0}{\frac{-[1-C'(1-T')]M}{[L_i(1-C'(1-T'))+I'L_Y]P^2}}}dP +$$

$$\boxed{\underset{>0}{\frac{I'L_Y}{L_i(1-C'(1-T'))+I'L_Y}}}d\pi_e + \boxed{\underset{>0}{\frac{-L_Y}{L_i(1-C'(1-T'))+I'L_Y}}}dG$$

Plug back into dY equation to get dY multipliers:

$$dY = \boxed{\underset{>0}{\frac{I'}{[L_i(1-C'(1-T'))+I'L_Y]P}}}dM + \boxed{\underset{<0}{\frac{-I'M}{[L_i(1-C'(1-T'))+I'L_Y]P^2}}}dP +$$

$$\boxed{\underset{>0}{\frac{-I'L_i}{L_i(1-C'(1-T'))+I'L_Y}}}d\pi_e + \boxed{\underset{>0}{\frac{L_i}{L_i(1-C'(1-T'))+I'L_Y}}}dG$$

Note, the terms for dπ_e and dG have two terms that need to be combined. The math is ugly, but easy. Multipliers above show final result.

Extra Multipliers - note that you can get additional multipliers from $dY = C'(1 - T_Y')dY + I'di - I'd\pi_e + dG$ by recognizing that $dC = C'(1 - T')dY$

$$dC = \boxed{\begin{array}{c}\dfrac{C'(1-T')I'}{[L_i(1-C'(1-T'))+I'L_Y]P} \\ > 0\end{array}} dM + \boxed{\begin{array}{c}\dfrac{-C'(1-T')I'M}{[L_i(1-C'(1-T'))+I'L_Y]P^2} \\ < 0\end{array}} dP +$$

$$\boxed{\begin{array}{c}\dfrac{-C'(1-T')I'L_i}{L_i(1-C'(1-T'))+I'L_Y} \\ > 0\end{array}} d\pi_e + \boxed{\begin{array}{c}\dfrac{C'(1-T')L_i}{L_i(1-C'(1-T'))+I'L_Y} \\ > 0\end{array}} dG$$

You can also get additional multipliers form the di equation by recognizing $dI = I'di - I'd\pi_e$

$$dI = \boxed{\begin{array}{c}\dfrac{I'[1-C'(1-T')]}{[L_i(1-C'(1-T'))+I'L_Y]P} \\ > 0\end{array}} dM + \boxed{\begin{array}{c}\dfrac{-I'[1-C'(1-T')]M}{[L_i(1-C'(1-T'))+I'L_Y]P^2} \\ < 0\end{array}} dP +$$

$$\boxed{\begin{array}{c}\dfrac{-I'[1-C'(1-T')]L_i}{L_i(1-C'(1-T'))+I'L_Y} \\ > 0\end{array}} d\pi_e + \boxed{\begin{array}{c}\dfrac{-I'L_Y}{L_i(1-C'(1-T'))+I'L_Y} \\ < 0\end{array}} dG$$

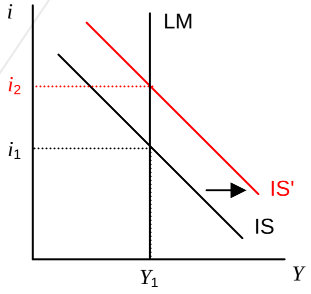
Looking at Changes - take dY/dG...

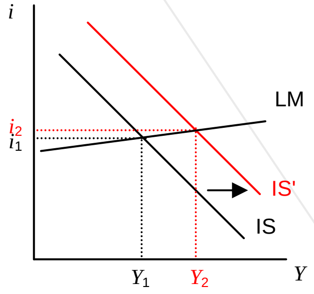
No Money: $\dfrac{1}{1-C'(1-T')}$ > **With Money**: $\dfrac{L_i}{L_i(1-C'(1-T'))+I'L_Y}$

No money version treats LM curve as flat (horizontal); says demand for money is sensitive to interest rates; with money, multiplier is smaller because G↑ causes i↑ which causes Y↓ so the overall change in Y is less than it was before considering the money market

Sensitive to L_i - note if $L_i = 0$ (i.e., demand for money is insensitive to interest rates), LM curve is vertical; increase in G leaves Y unchanged and increases i

Implication for Fiscal Policy - have to worry about how sensitive money demand is to interest rate (L_i); larger L_i means fiscal policy is more effective (i.e., greater change in Y with less impact on i)





Recap - looked at model three times:

$$\frac{dY}{dG} = \frac{1}{1-C'} > \frac{1}{1-C'(1-T')} > \frac{L_i}{L_i\left(1-C'(1-T')\right)+I'L_Y}$$

(1) T & I exogenous (given) (2) I exogenous & T endogenous (3) T & I endogenous

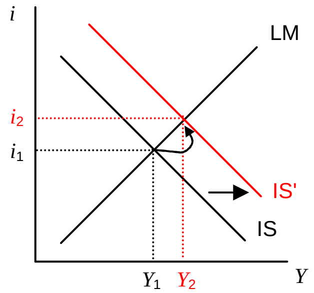
(1) Larger MPC (C') $\Rightarrow dY/dG$ larger
 Note: this conclusion is more important than the actual value of the multiplier

(2) Larger marginal tax rate (T') $\Rightarrow dY/dG$ smaller
 Note: conclusion form first model is still valid in the second. Start simple (or later purposely make things exogenous) to make conclusions more obvious.

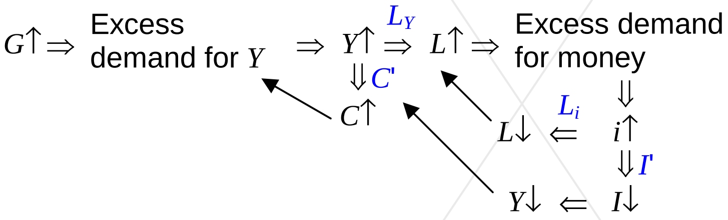
(3) Demand for money more sensitive to interest rate (larger $|L_i|$) $\Rightarrow dY/dG$ larger
 Note: $L_i = 0 \Rightarrow dY/dG = 0$ (see graph with vertical LM curve)
 Note: conclusions form first and second models still valid in third

More Results - go back to T being exogenous (i.e., $T' = 0$); this simplifies the multipliers to find other conclusions:

$$\frac{dY}{dG} = \frac{L_i}{L_i(1-C')+I'L_Y} > 0$$



Change in *G* - Short Version:

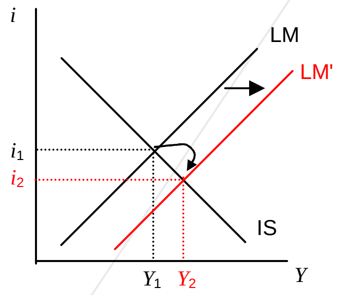


Long Version - increase in G causes too much demand for goods (i.e., excess demand for Y); firms increase output to eliminate excess demand ($Y\uparrow$); as firms increase output (1) C increases (based on C', marginal propensity to consume or the sensitivity of consumption to income) and (2) demand for money (L) increases (based on L_Y, sensitivity of demand for money to income); increased C further increases demand for goods (this goes back to the original dY/dG multiplier) while increased demand for money forces interest rates (i) to climb (based on L_i, sensitivity of demand for money to interest rates); higher rates eliminate excess demand for money (pulls it back to LM curve) and increase borrowing costs for investment so I decreases (based on I', sensitivity of investment to interest rate); process continues until decrease in I soaks up excess demand ("**crowding out**")

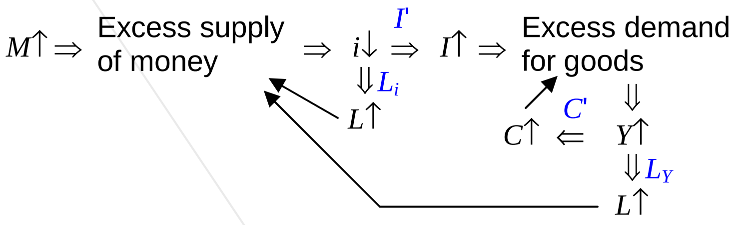
Analyzing Components -

C' larger $\Rightarrow dY/dG$ larger
L_Y larger $\Rightarrow dY/dG$ smaller
$|L_i|$ larger $\Rightarrow dY/dG$ larger
I' larger $\Rightarrow dY/dG$ smaller

$$\frac{dY}{dM} = \frac{I'}{[L_i(1-C')+I'L_Y]P} > 0$$



Change in M - Short Version:

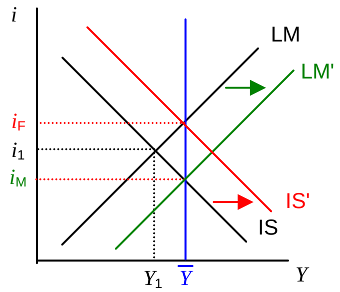


Long Version - increase in M causes too much supply of money which drops interest rates (i); lower rates increase demand for money ($L\uparrow$, based on L_i) and increase investment ($I\uparrow$, based on I'); increased investment increases demand for goods so output increases; this increases consumption ($C\uparrow$, based on C') and increases the demand for money ($L\uparrow$, based on L_Y); eventually the increased demand for money from lower interest rate and increased output will offset the excess supply

Analyzing Components -

C' larger $\Rightarrow dY/dM$ larger (same as with dY/dG)
L_Y larger $\Rightarrow dY/dM$ smaller (same as with dY/dG)
$|L_i|$ larger $\Rightarrow dY/dM$ smaller (opposite of with dY/dG)
I' larger $\Rightarrow dY/dM$ larger (opposite of with dY/dG)

Monetary or Fiscal Policy? -



- **Multipliers** - size of multipliers not important unless one of them is zero because you can always get to potential GDP
- **Conventional Wisdom** - monetary is better because it's easier to change M than G (less political)
- **Interest Rates** - both policies get to potential GDP, but fiscal policy increases interest rates and monetary policy lowers interest rates
- **Composition of Output** - $Y = C + I + G < \overline{Y}$; in order to increases Y, one of the components has to increase; fiscal and monetary policy target different components; fiscal policy through government purchases increases G; fiscal policy through taxes increases C; in both cases, fiscal policy results in decreased I; monetary policy increases I
- **Future Growth** - recall Solow Model said I (savings) has consequences for growth

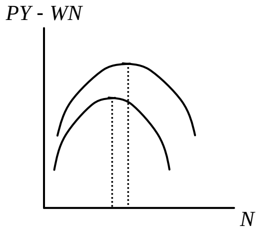
Problem with "Fact" 3? - said economy would fix itself, but we've only looked at government intervention through monetary or fiscal policy; how does economy fix itself? If we're below potential we know we have unemployment, but we ignored the labor market;

Prices - we also kept prices constant throughout; that will change when we add the labor market

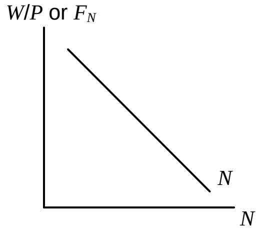
IS-LM Model with Labor

Demand for Labor (N^D) - assumes firms hire workers to make profit

- **Profit** - price x quantity [i.e., total revenue] - (labor cost + materials cost + fixed cost) [i.e., total cost]
- **Value Added** - since we're talking about GDP, we're only worried about value added; that means we can ignore raw materials cost; we also change price to a "value added" or "net" price (P) to account for the raw materials
- **Labor Decision** - fixed costs are irrelevant in the decision; use net price (P) times output (Y) minus wage (W) times number of workers (N): **Profit = $PY - WN$**
- **Assumptions** -
  - (1) competitive goods and labor markets (i.e., can't pick P or W)
  - (2) capital (K) is fixed in the short run
  - (3) production function relates Y to N: $Y = F(K,N)$
  - (4) production function is increasing at diminishing rate ($F_N > 0$; $F_{NN} < 0$)
- **Hiring Decision** - want to maximize $PF(K,N) - WN$
  - **1st Order Condition** - set derivative to zero: $PF_N - W = 0$
  - **2nd Order Condition** - $d^2(PY - WN)/dN^2 = F_{NN} < 0$ (by assumption)
  - **Marginal Product of Labor (MPL)** - F_N
  - **Marginal Revenue of Labor** - PF_N
  - **Marginal Cost of Labor** - W



- **Shifting Curve** - three things will shift curve and change hiring decision:
  - (1) **W** - $W\uparrow$ moves curve down; hire fewer workers
  - (2) **P** - $P\uparrow$ moves curve up; hire more workers
  - (3) **Technology** - tech improvement increases F_N; hire more workers
- **MPL = Real Wage** - rearrange $PF_N = W$ to get MPL = Real Wage (w):
  $F_N = W/P = w$
- **Demand Curve** - since firms are willing to pay the MPL, the MPL is basically the demand curve for labor
  - **Downward Sloping** - $d(W/P)/dN = d(F_N)/dN = F_{NN} < 0$ (for all production functions with diminishing returns)



Supply of Labor ($\overline{N}$) - in short run, only so many workers so supply is fixed; besides, empirical studies show the supply of labor is inelastic

Classical Model (Long Run) - always have full employment; more appropriate for long run; describes aspects we still worry about, but doesn't capture it all (that's why we have Keynesian Model)

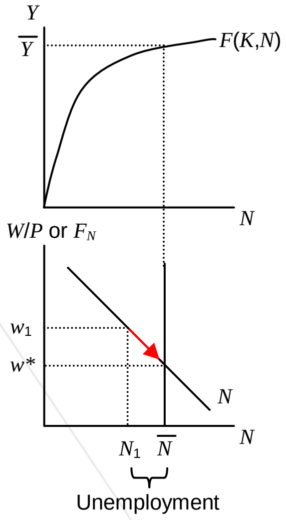
- **Full Employment** - if real wage is too high (w_1), there is downward pressure on wages to move toward equilibrium in the labor market; eventually everyone is employed
- **5 Equations** - combining these 3 new equations with the IS-LM model gives us long term equilibrium; unknowns are Y, i, w, P, N

Labor Market $\begin{cases} F_N = w \\ N = \overline{N} \\ Y = F(K,N) \end{cases}$

Goods Market $Y = C(Y - T) + I(i - \pi_e) + G$

Asset Market $L(Y, i) = M/P$

Note: Really have 7 equations & 7 unknowns because C and I are embedded in the model; could have 8 if you make taxes endogenous



Get Multipliers - take total differentials

(1) $F_{NN}\,dN + F_{NK}\,dK = dw$

(2) $dN = d\overline{N}$

(3) $dY = F_K\,dK + F_N\,dN$

(4) $dY = C'dY - C'dT + I'di - I'd\pi_e + dG$

(5) $L_Y\,dY + L_i\,di = dM/P - (M/P^2)dP$

Others - if you want $dC/d?$ and $dI/d?$, you'll need $\mathbf{dC = C'dY - C'dT}$ and $\mathbf{dI = I'di + I'd\pi_e}$

Reduced Form - get one endogenous (unknown) variable on the left side with all exogenous (known) variables on the other

$dN = \boxed{1}\,d\overline{N}$ (from Eqn (2)); not a very exciting result, but realize $dN/d? = 0$ for all other exogenous variables.

Plug this result into dY from Eqn (3):

$dY = \boxed{F_K}\,dK + \boxed{F_N}\,d\overline{N}$; (both > 0) very surprising result... back to the Solow Model; notice the only things that affect output (Y) in the long run are capital (K) and labor (N);

Fiscal Policy - $dY/dG = 0$; makes since in long run because if economy is at full employment, government purchases don't impact the number of workers or the capital base; potential output remains constant; if you calculate all the multipliers, you can also see that $di/dG = -1/I'$ and $dI/dG = -1$ (**100% crowding out**); $dY/dT = 0$; lowering taxes doesn't change affect supply (still only K and N that do that); total output doesn't change, but composition of the output changes because consumption goes up and investment goes down; $dI/dT = C'$ (100% crowding out)

Monetary Policy - $dY/dM = 0$; you can also calculate $dI/dM = 0$ and $di/dM = 0$; changing M only affects one of the endogenous variables: P; $dP/dM = P/M > 0$; this means increasing money supply just increases prices in proportion

Summary - in long run, fiscal policy just changes division of output between C, I, and G; monetary policy just changes price level (inflation); remember, the only thing that increases potential output is capital and labor (and technology improvement)

Plug dN into Eqn (1):

$dw = \boxed{F_{NN}}\,d\overline{N} + \boxed{F_{NK}}\,dK$

Plug in dY into Eqn (4) and solve for di:

$$di = \boxed{\begin{array}{c}\dfrac{(1-C')F_K}{I'}\\ <0\end{array}}\,dK + \boxed{\begin{array}{c}\dfrac{(1-C')F_N}{I'}\\ <0\end{array}}\,d\overline{N} + \boxed{\begin{array}{c}\dfrac{C'}{I'}\\ <0\end{array}}\,dT + \boxed{\begin{array}{c}1\\ >0\end{array}}\,d\pi_e + \boxed{\begin{array}{c}\dfrac{-1}{I'}\\ >0\end{array}}\,dG$$

Plug in dY and di into Eqn (5) and solve for dP:

$(M/P^2)\text{dP} = dM/P - L_Y\,dY - L_i\,di =$

$$(M/P^2)dP = dM/P - L_YF_K\,dK - L_YF_N\,d\overline{N} - \frac{(1-C')F_KL_i}{I'}dK - \frac{(1-C')F_NL_i}{I'}d\overline{N} - \frac{C'L_i}{I'}dT - L_i\,d\pi_e - \frac{-L_i}{I'}dG$$

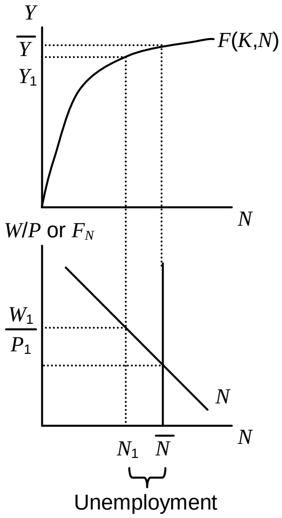
$$dP = \boxed{\begin{array}{c}\frac{P}{M}\\ >0\end{array}}\, dM + \boxed{\begin{array}{c}\frac{-P^2(L_i(1-C')+I'L_Y)F_K}{MI'}\\ <0\end{array}}\, dK + \boxed{\begin{array}{c}\frac{-P^2(L_i(1-C')+I'L_Y)F_N}{MI'}\\ <0\end{array}}\, d\overline{N} +$$

$$\boxed{\begin{array}{c}\frac{-P^2C'L_i}{MI'}\\ <0\end{array}}\, dT + \boxed{\begin{array}{c}\frac{-P^2L_i}{M}\\ >0\end{array}}\, d\pi_e + \boxed{\begin{array}{c}\frac{-P^2L_i}{MI'}\\ >0\end{array}}\, dG$$

Other Multipliers - use $dC = C'dY - C'dT$ and $dI = I'di - I'd\pi_e$ to get other multipliers

$$dC = \boxed{\begin{array}{c}C'F_K\\ >0\end{array}}\, dK + \boxed{\begin{array}{c}C'F_N\\ >0\end{array}}\, d\overline{N} + \boxed{\begin{array}{c}-C'\\ <0\end{array}}\, dT$$

$$dI = \boxed{\begin{array}{c}(1-C')F_K\\ >0\end{array}}\, dK + \boxed{\begin{array}{c}(1-C')F_N\\ >0\end{array}}\, d\overline{N} + \boxed{\begin{array}{c}C'\\ >0\end{array}}\, dT + \boxed{\begin{array}{c}-1\\ <0\end{array}}\, dG$$



Keynesian Model (**Short Run, Fixed-Wage**) - Not always operating with equilibrium in all three markets; short run model looks at what happens in reaching equilibrium; usual assumption is that wage is too high to have equilibrium in the labor market ("excess supply" = unemployment... economy is on the demand curve for labor, but not on the supply curve)

Goods Market - $C + I + G - Y$ = excess demand; if = 0, we have goods market equilibrium; if > 0, there is excess demand and prices rise; if < 0, there is excess supply and prices fall; $\therefore a(C + I + G - Y) = dP/dt$ (change in price with respect to time)

Asset Market - similar argument; $b(L - M/P) = di/dt$ (change in interest rates with respect to time)

Labor Market - similar argument; $c(ND - N) = dW/dt$ (change in wage with respect to time

Constants - a, b, and c determine how fast prices, interest rates, and wages change to bring things back to equilibrium; of the three, interest rates are quickest (almost instantly); wages are the slowest to respond

Simulation - we could simulate to see the in between effects, but need to know all the equations (i.e., explicit functions) and results wouldn't be general

Stepped Comparative Statics - instead of simulating, look at an in between static condition were we get to goods and asset equilibrium, but not labor (since it's slowest to respond); $\therefore$we look at ***W* being fixed** and drop Eqn (2) from the long run model; we also have to use W/P rather than w

4 Equations - unknowns are Y, i, P, N

Labor Market $\begin{cases} F_N = W/P \\ Y = F(K,N) \end{cases}$

Goods Market $Y = C(Y - T) + I(i - \pi_e) + G$

Asset Market $L(Y, i) = M/P$

Note: Really have 6 equations & 6 unknowns because C and I are embedded in the model; could have 7 if you make taxes endogenous

Get Multipliers - take total differentials

(1) $F_{NN}dN + F_{NK}dK = dW/P + (W/P^2)dP$

(2) $dY = F_K dK + F_N dN$

(3) $dY = C'dY - C'dT + I'di - I'd\pi_e + dG$

(4) $L_Y dY + L_i di = dM/P - (M/P^2)dP$

Work done in Homework 2

$$dY = \boxed{\begin{array}{c}\dfrac{(F_{NN}F_K - F_{NK}F_N)I'M}{Pz}\\ > 0\end{array}}\, dK + \boxed{\begin{array}{c}\dfrac{-F_N I'M}{P^2 z}\\ < 0\end{array}}\, dW + \boxed{\begin{array}{c}\dfrac{-(F_N)^2 L_i C'}{z}\\ < 0\end{array}}\, dT \;+$$

$$\boxed{\begin{array}{c}\dfrac{-(F_N)^2 I' L_i}{z}\\ > 0\end{array}}\, d\pi_e \;+ \boxed{\begin{array}{c}\dfrac{(F_N)^2 L_i}{z}\\ > 0\end{array}}\, dG + \boxed{\begin{array}{c}\dfrac{(F_N)^2 I'}{Pz}\\ > 0\end{array}}\, dM$$

$$di = \boxed{\begin{array}{c}\dfrac{-(1-C')(F_{NN}F_K - F_{NK}F_N)M}{Pz}\\ < 0\end{array}}\, dK + \boxed{\begin{array}{c}\dfrac{-(1-C')F_N M}{P^2 z}\\ > 0\end{array}}\, dW +$$

$$\boxed{\begin{array}{c}\dfrac{C'\left[(F_N)^2 L_Y - F_{NN}M/P\right]}{z}\\ < 0\end{array}}\, dT + \boxed{\begin{array}{c}\dfrac{I'(L_Y(F_N)^2 - F_{NN}M/P)}{z}\\ > 0\ (< 1)\end{array}}\, d\pi_e +$$

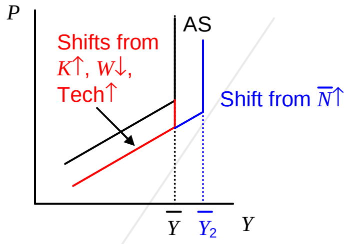
$$\boxed{\begin{array}{c}\dfrac{F_{NN}M/P - L_Y(F_N)^2}{z}\\ > 0\end{array}}\, dG + \boxed{\begin{array}{c}\dfrac{(1-C')(F_N)^2}{Pz}\\ < 0\end{array}}\, dM$$

$$dP = \boxed{\begin{array}{c}\dfrac{PF_{NN}L_i C'}{z}\\ < 0\end{array}}\, dT + \boxed{\begin{array}{c}\dfrac{I'PF_{NN}L_i}{z}\\ > 0\end{array}}\, d\pi_e + \boxed{\begin{array}{c}\dfrac{-PF_{NN}L_i}{z}\\ > 0\end{array}}\, dG + \boxed{\begin{array}{c}\dfrac{-I'F_{NN}}{z}\\ > 0\end{array}}\, dM +$$

$$\boxed{\begin{array}{c}\dfrac{F_N(L_i(1-C') + I'L_Y)}{z}\\ > 0\end{array}}\, dW + \boxed{\begin{array}{c}\dfrac{P(F_{NN}F_K - F_{NK}F_N)(L_i(1-C') + I'L_Y)}{z}\\ ?\end{array}}\, dK$$

$$\boxed{z = (F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P < 0}$$

$F_{NN}F_K$ - $F_{NK}F_N$ - this term is negative for most production functions (including all Cobb-Douglas production functions); the first term is negative by the diminishing returns assumption ($F_{NN} < 0$); if the second (F_{NK}) is nonnegative, the difference is negative; in order for the difference to be positive must have $F_{NK} < F_{NN}F_K/F_N$ (which is negative)



Aggregate Supply and Demand

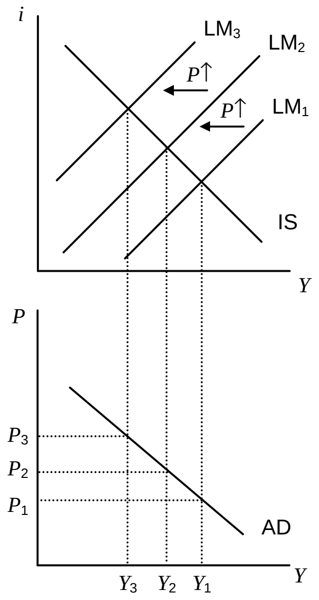
Aggregate Supply (AS) - first two equations for short-run model:

Labor Market $\left\{\begin{array}{l} F_N = W/P \\ Y = F(K,N) \end{array}\right.$

2 equations and 3 unknowns (Y, P, N); if you graph all combinations of Y and P that solve the system for a given level of N, you end up with the AS curve

Potential GDP (pGDP) - AS curve is vertical at this point; doesn't matter how much higher price level gets, firms can't supply any more

Shifts in AS - AS↑ (shift right) if $K\uparrow$, $W\downarrow$, or Technology advances; pGDP shifts right (along with AS) if $\bar{N}\uparrow$



Aggregate Demand (AD) - substitution effect and income effect result form change in relative prices; this model talks about price level (i.e., all goods) so it has no effect on aggregate demand; instead, look at last two equations for short-run model:

Goods Market $Y = C(Y - T) + I(i - \pi_e) + G$

Asset Market $L(Y, i) = M/P$

2 equations and 3 unknowns (Y, P, i); if you graph all combinations of Y and P that solve the system, you end up with the AD curve

LM Curve - price enters in $L(Y, i) = M/P$... $P\uparrow \Rightarrow$ LM↓ $\Rightarrow Y\downarrow$ (see graph)

AD Slope - < 0; get this result form IS-LM multiplier for dY/dG

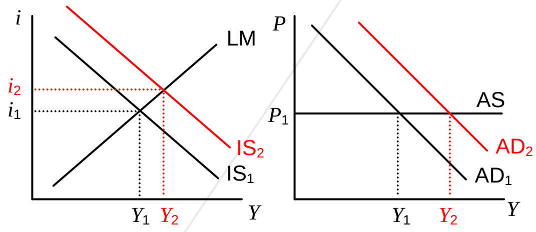
Shifts in AD - AD↑ (shift right) if $G\uparrow$, $T\downarrow$, or $\pi_e\uparrow$ (i.e., IS↑); or $M\uparrow$ (i.e., LM↑)

Fiscal Policy - trying to get back to pGDP by $G\uparrow$ or $T\downarrow$ (i.e., IS↑)

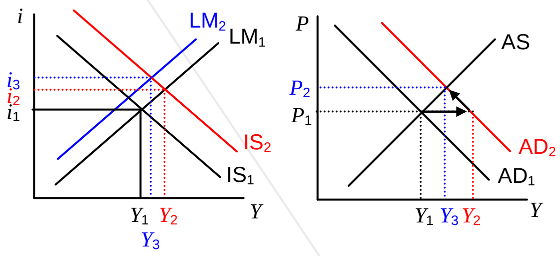
Fixed-Price Multiplier - assumes AS is horizontal line; this ignores change in price

Short-Run Multiplier - $P\uparrow \Rightarrow$ LM↓ $\Rightarrow i\uparrow$ and $y\downarrow$; so ΔY will be less and Δi will be more than fixed-price multiplier predicted

Fixed-Price Multiplier



Short-Run Multiplier
ΔY smaller; Δi larger

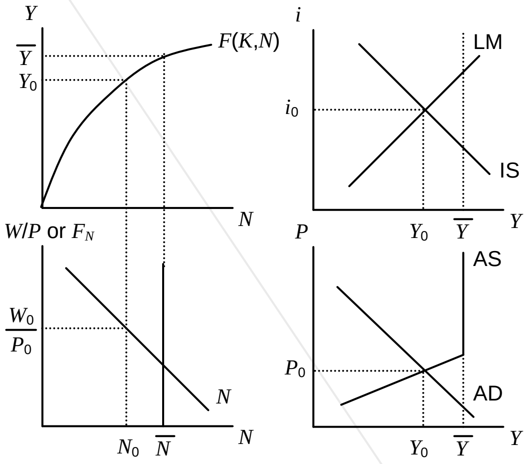


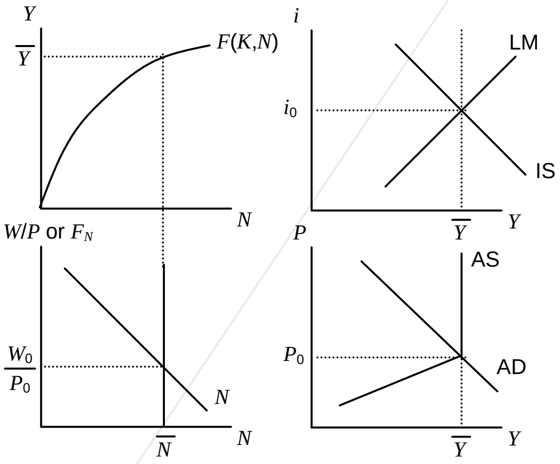
Monetary Policy - similar result except LM shifts twice (first LM↑ from $M\uparrow$; then LM↓ from $P\uparrow$); similar result: ΔY smaller; Δi smaller (than with fixed-price multiplier)

Policy Implications - if you do nothing, wages fall and AS↑; get to pGDP in deflationary way; if you use fiscal policy, AD↑ so you get to pGDP faster, but in inflationary way

Putting It All Together

Short-Run vs. Long-Run

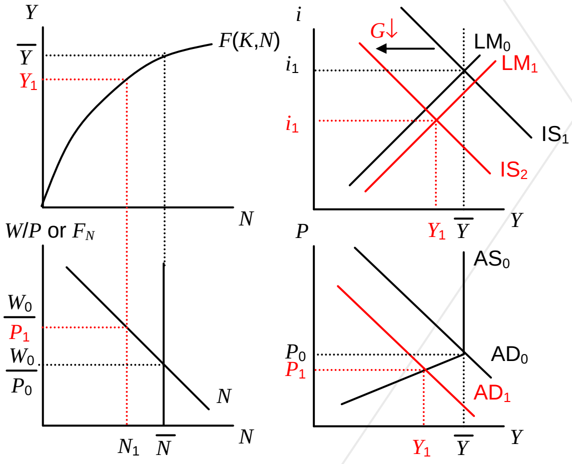


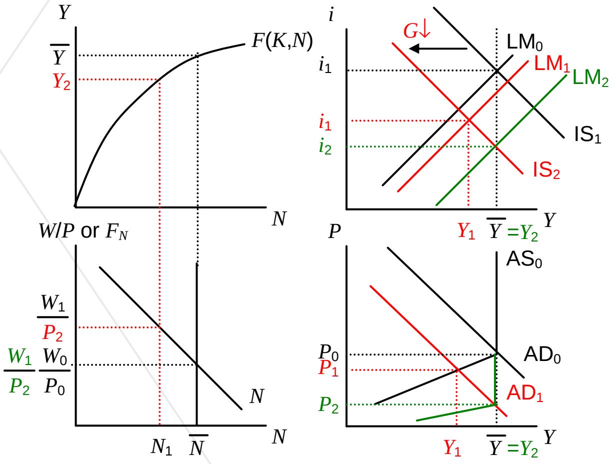


Fiscal Policy - assume at pGDP and look at $G\downarrow$

Short-Run vs. Long-Run

$G\downarrow \Rightarrow \text{IS}\downarrow \Rightarrow \text{AD}\downarrow \Rightarrow P\downarrow \Rightarrow \text{LM}\uparrow$ $\quad$ $\text{W}\downarrow \Rightarrow \text{AS}\uparrow \Rightarrow P\downarrow \Rightarrow \text{LM}\uparrow$





dY/dG - fixed-price > short-run > long-run = 0

di/dG - fixed-price < short-run < long-run

dP/dG - fixed-price < short-run < long-run

Result - although ultimately the value of Y doesn't change in the long-run, it's composition does change; we're trading off G for I (i.e., if $G\uparrow$, then $I\downarrow$ by same amount in long-run); if using taxes, $T\downarrow \Rightarrow C\uparrow \Rightarrow I\downarrow$ by same amount in long-run

Monetary Policy - assume at pGDP and have $M\downarrow$; same as above except first shift is LM↓ instead of IS↓; everything else is similar

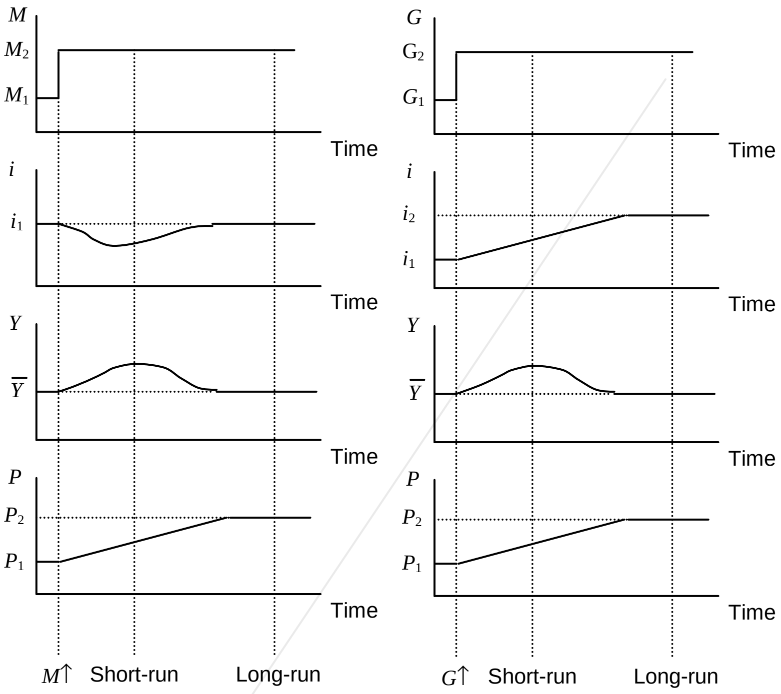
dY/dM - fixed-price > short-run > long-run = 0

di/dM - fixed-price < short-run < long-run = 0

dP/dM - fixed-price < short-run < long-run = P/M

Neutral Money - composition of Y is unchanged (in terms of C, I, and G), unlike with fiscal policy

Timing - so when is short-run and long-run?

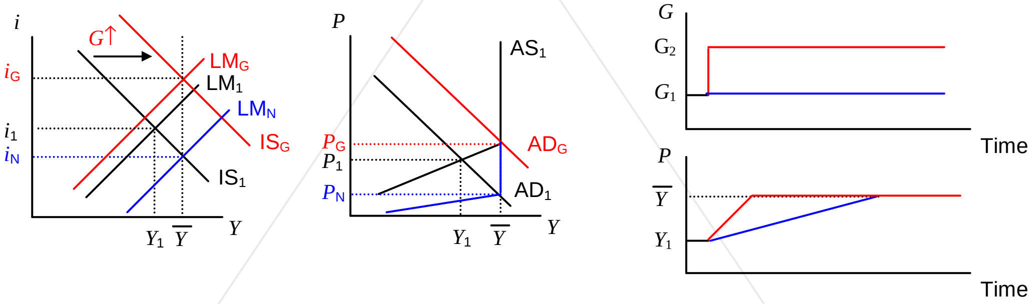


Fiscal Policy vs. **Do Nothing** - can use short-run multipliers to figure out exactly how much ΔG it will take to go from Y_1 to $\overline{Y}$ (by accounting for $P\uparrow \Rightarrow$ LM$\downarrow$); if there were no fiscal policy, eventually $W\downarrow \Rightarrow$ AS$\uparrow$ & LM$\uparrow$ and you get to same result; difference is fiscal policy is faster at expense of higher i which means less I

Monetary Policy - end up with same i as if economy fixed itself, but have higher P; prevents deflation that would've occurred naturally

Long-Run - dY/dG = 0... compare Y in long-run after $G\uparrow$ to what Y would've been without $G\uparrow$; it's the same

Capital Stock - ignored it in IS-LM (covered in Solow Model); although it takes a long time to be evident (very long-run), $G\uparrow$ decreases growth rate of $\overline{Y}$ (less investment)



Other Areas to Explore -

(1) Potential Output = "full employment"; but how to we define this?

(2) How do we modify IS-LM to deal with ongoing inflation; what about ongoing increase in money supply?

(3) How are expectations formed? (π_e)

(4) How do IS-LM results change for open economy

(5) **Consumption** - considered it a function of after tax income: $C(Y - T)$... will study in next section

IS-LM Model with Open Economy

Why Waited - most closed economy results follow through so it was easier to get them with closed economy

Currencies

	Dollar
Switzerland	1.3161

	SFranc
U.S.	0.75980

Cross Rates - e.g. from WSJ:

Two was to present same thing; first one is from point of view of buyer of dollars (i.e., 1.3161 Swiss francs per dollar); second one is from point of view of buyer of Swiss franks (i.e., \$0.75980/SFranc); Note: this numbers are reciprocals: 1/1.3161 = 0.75980

Analogy - usually convention for prices looks from point of view of person with paper in hand (i.e., dollars); buying gas is priced in \$/gal (e.g., \$1.50/gal); from point of view of gas station though, they're exchanging gal/\$ (e.g., 2/3 gal/\$)

Two Country Model - look at home country vs. foreign country (rest of the world)

Exchange Rate (E) - price of foreign currency in domestic currency (e.g., if U.S. is the home country, use E = \$0.75980/SFranc); E with worthless unless you know who is the host country (and foreign country)

\$ Depreciates - $E\uparrow \Rightarrow$ value of dollar↓; takes more to buy a SFranc

\$ Appreciates - $E\downarrow \Rightarrow$ value of dollar↑; takes fewer to by a SFranc

Real Exchange Rate (e) - have to account for price levels in both countries: $\boldsymbol{e = EP^*/P}$

$\boldsymbol{P}$ - price level in home country

$\boldsymbol{P^*}$ - price level is foreign country

Example - if P^* is constant and both E and P double, there is effectively no difference on trade; from point of view of Swiss, U.S. goods costs the same: they're price is twice as high, but they can buy twice as many dollars for each SFranc; from point of view of U.S., Swiss goods effectively double in price because of E, but U.S. goods also doubled in price

Floating Exchange Rate - governments don't try to maintain a certain level; E free to adjust

Trade

Supply-Demand - $Y = C + I + G$; not complete

Exports (EX) - goods made in home country, but bought somewhere else; captured in Y, but not in $(C + I + G)$

Imports (IM) - goods bought in home country, but made somewhere else; captured in $(C + I + G)$, but not in Y

Net Exports (NX) - exports minus imports; $NX = EX - IM$

Function - $NX = X(e, Y - T, Y^* - T^*)$

e - real exchange rate tells if goods are relatively more or less expensive in home or foreign country; $e\uparrow$ means home currency weaker so imports are more expensive and exports are cheaper (i.e., $NX\uparrow$)

$X_e > 0$	$e\uparrow \Rightarrow NX\uparrow$
$X_Y < 0$	$(Y - T)\uparrow \Rightarrow NX\downarrow$
$X_{Y^*} > 0$	$(Y^* - T^*)\uparrow \Rightarrow NX\uparrow$

$Y - T$ - disposable income in home country; more income means you can afford more imports $\therefore (Y - T)\uparrow \Rightarrow NX\downarrow$

$Y^* - T^*$ - disposable income in foreign country; more income means they can afford more of home country's exports $\therefore (Y^* - T^*)\uparrow \Rightarrow NX\uparrow$

Completed - $Y - EX = C + I + G - IM \Rightarrow Y = C + G + G + EX - IM \Rightarrow \boldsymbol{Y = C + I + G + NX}$

Trade Deficit - Imports > Exports (i.e., $NX < 0$)

Drag on Demand - if disposable income increases, $Y\uparrow$ from MPC (C'), but it's offset by marginal propensity to import (X_Y); this is a drag on demand, but trade deficit has more important impact in capital flows

Equilibrium? - if imports "sell home currency" and exports "buy home currency", how is it possible to have a trade deficit or surplus? Capital flows are actually more important than trade in currency markets

Capital Flows

Capital Inflows - foreign investment in home country assets (land, firms, gov't securities, etc.)

Capital Outflows - home country investment in foreign assets

Net Capital Inflow (CF) - capital inflows - capital outflows

Supply-Demand for Currency - have supply and demand for home country currency; price of currency is E; supply is determined by exports plus capital inflows; demand is determined by imports plus capital outflows

Equilibrium - supply = demand $\Rightarrow EX + CI = IM + CO \Rightarrow (EX - IM) + (CI - CO) = 0 \Rightarrow$

$NX + CF = 0$

Interest Rates - difference between home country interest rate (i) and foreign country interest rate (i^*) influence CF; if $(i - i^*) > 0$, home country has higher interest rates so that would attract more net investment (i.e., $CF > 0$); relative changes from equilibrium:

$(i - i^*)\uparrow \Rightarrow CF\uparrow \Rightarrow e\downarrow \Rightarrow NX\downarrow$; this is why $e(i - i^*)$ has $e' < 0$

Saving - income you don't spend

Government Saving (S_G) - net taxes minus government purchases; $S_G = T - G$; negative for any government running a budget deficit

Private Saving (S_P) - after-tax income minus consumption; $S_P = Y - T - C$

Substitute $Y = C + I + G + NX$... $S_P = C + I + G + NX - T - C$

Rearrange to get I by itself... $I = S_P + T - G - NX$

Realize $S_G = T - G$ and $NX = -CF$... $I = S_P + S_G + CF$

Interpretation - investment is based on private saving, government saving, and net capital inflow (saving from rest of world available to the home country)

Example - suppose $G\uparrow$ by 100:	I =	S_P +	S_G +	CF	
Closed Economy	-100	0	-100	n/a	
Closed w/ $C(Y - T, i - \pi_e)$	-90	10	-100	n/a	$C_i < 0$ so $i\uparrow \Rightarrow C\downarrow$ (i.e., $S_P\uparrow$)
Ricardian	-80	20	-100	n/a	$G\uparrow$ with no $\Delta T \Rightarrow S_P\uparrow$
Open	-60	0	-100	+40	

Since $i\uparrow$ in home country, S_P^* (foreign saving) is drawn to that country through CF

Trade Deficit Revisited - common explanations for trade deficit include trade barriers, interest rates, exchange rates, etc., but root cause is savings: $I = S_P + S_G + CF$

Case 1 - Rich vs. Poor - expect capital flows from rich countries to poor countries because the less developed countries have better investment opportunities (assuming stable political environment); rich country lends to poor country and $CF > 0$ for poor country; so $NX < 0$ (i.e., trade deficit)

Example - UK vs. US during 1800s

Case 2 - Rich vs. Rich - rich home country with lower S_P than rich foreign country; the foreign country's higher savings result in lower i so expect capital flows from high S_P country to low S_P country

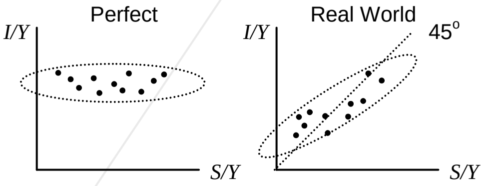
Example - US vs. Japan during 1980s

Imperfect Capital Flows - barriers or rules restrict capital flow

Perfect Capital Flows - slightest difference in interest rates bring huge capital inflow and large change in e and NX; $e' \rightarrow -\infty$

Result - implies that i would be the same everywhere; IS curve would be horizontal at same level for every country; if $G\uparrow$, home country borrows more ($i\uparrow$); money comes from everywhere so there is very little crowding out

Why $i \neq i^*$ - risk; expected changes in e



Investment vs. Savings - look at graph of investment as percentage of GDP (I/Y) and savings as percentage of GDP (S/Y); if there is perfect capital flow, there should be no relationship (i.e., slope zero); if there is a relationship that means people invest more domestically rather than looking for higher I in foreign country

45^o Line - denotes were savings equals investment; countries below the line have more saving than investment so they are net lenders of money (i.e., trade surplus); countries above the 45o line are net borrowers (i.e., trade deficit)

Endogenous vs. Exogenous i^* - out model assumes $G\uparrow$ in home country doesn't affect i^* (i.e., exogenous); to account for the changes (i.e., make it endogenous), we have to add equations for foreign country: $Y^* = C^* + I^* + G^* + NX^*$, etc.

Why Not - (1) it's hard; (2) other results still good; (3) if home country is small, $G\uparrow$ doesn't impact the rest of the world much so exogenous i^* is good enough (small country model)

IS-LM Model with Open Economy

Bomberger - "Romer suffers from an excess in generality"

Our Model - floating exchange rate, imperfect capital mobility, & small country

Basically adding 3 equations and 3 unknowns to the IS-LM model (short-run or long-run)

Equations	**Differentials**
$NX = X(e, Y - T, Y^* - T^*)$	$dNX = X_e\,de + X_Y\,dY - X_Y\,dT + X_{Y^*}dY^* - X_{Y^*}dT^*$
$e = e(i - i^*)$	$de = e'di - e'di^*$
$e = EP^*/P$	$de = (P^*/P)dE + (E/P)dP^* - (EP^*/P^2)dP$

Also modify: $Y = C(Y - T) + I(i - \pi_e) + G + NX$

Long-Run Multipliers - more important than short-run and easier to calculate

Monetary Policy (ΔM) - end result is same as closed economy:

$dP/dM = P/M > 0$; $dY/dM = di/dM = dI/dM = 0$

New Terms -

$dE/dM = E/M$

$de/dM = 0$... $e = EP^*/P$; ΔE countered by ΔP so real exchange rate doesn't change (e.g., $M\uparrow$ 10% $\Rightarrow$ $P\uparrow$ 10% and $E\uparrow$ 10%)

$dNX/dM = 0$... follows from no change in real exchange rate

Impact - monetary policy has no long-run effect other than to increase the price level; the higher prices are offset in domestic and world markets by an increase in nominal exchange rates so real exchange rates remain unchanged (i.e., foreign and domestic products effectively cost the same relative to each other)

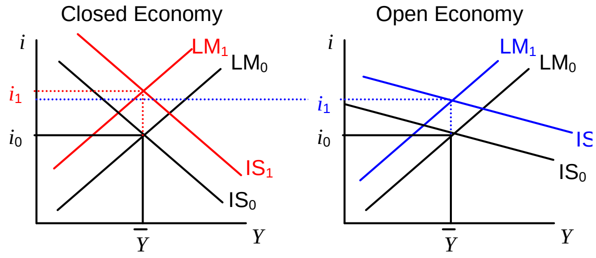
Fiscal Policy (ΔG) - with open economy IS curve is flatter so interest rates don't rise as much and there isn't as much crowding out as there is in a closed economy

Multiplier	**Closed Economy**	**<> (abs)**	**Open Economy**
$\frac{dY}{dG}$	0	=	0
$\frac{di}{dG}$	$\frac{-1}{I'} > 0$	>	$\frac{-1}{I'+X_e e'} > 0$
$\frac{dI}{dG}$	-1 < 0	>	$\frac{-I'}{I'+X_e e'} < 0$
$\frac{dP}{dG}$	$\frac{-P^2 L_i}{MI'} > 0$	>	$\frac{-P^2 L_i}{M(I'+X_e e')} > 0$
$\frac{dNX}{dG}$	n/a		$\frac{-X_e e'}{I'+X_e e'} < 0$

Example - Open $\quad$ Closed

$Y = C + I + G$ $\quad$ $Y = C + I + G + NX$

0 $\quad$ -100 +100 $\quad$ 0 $\quad$ -60 +100 -40



IS-LM Model Summary

Goods Market Only

Exogenous (fixed)	**Endogenous (2 unknowns)**	**Embedded Unknowns**
I	Y, C	none

Equations (2)	**Differentials**
$Y = C + I$	$dY = dC + dI$
$C = C(Y)$	$dC = C'dY$

Goods Market with Fiscal Policy

Exogenous (fixed)	**Endogenous (2 unknowns)**	**Embedded Unknowns**
I, G, T	Y, C	none

Equations (2)	**Differentials**
$Y = C + I + G$	$dY = dC + dI + dG$
$C = C(Y - T)$	$dC = C'dY - C'dT$

Goods Market Only with Taxes as Function of Income

Exogenous (fixed)	**Endogenous (2 unknowns)**	**Embedded Unknowns**
I, G	Y, C	T

Equations (2)	**Differentials**
$Y = C + I + G$	$dY = dC + dI + dG$
$C = C(Y - T(Y))$	$dC = C'(1 - T')dY$

Goods Market and Money Market (Endogenous Taxes)

Exogenous (fixed)	**Endogenous (2 unknowns)**	**Embedded Unknowns**
G, M, P, π_e	Y, i	C, I, T

Equations (2)	**Differentials**
$Y = C(Y - T(Y)) + I(i - \pi_e) + G$	$dY = C'(1 - T')dY + I'di - I'd\pi_e + dG$
$L(Y, i) = M/P$	$L_Y dY + L_i di = dM/P - (M/P^2)dP$

Note: For exogenous taxes, just plug in $T' = 0$ to all the multipliers

** All on this page could be considered **<u>Fixed Price</u>**

Goods Market, Money Market, and Labor Market (Long Run Equilibrium)

Exogenous (fixed)	Endogenous (5 unknowns)	Embedded Unknowns
G, M, π_e, T, K, $\overline{N}$	Y, i, w, P, N	C, I

Equations (5)	Differentials
$F_N = w$	$F_{NN}dN + F_{NK}dK = dw$
$N = \overline{N}$	$dN = d\overline{N}$
$Y = F(K,N)$	$dY = F_K dK + F_N dN$
$Y = C(Y - T) + I(i - \pi_e) + G$	$dY = C'dY - C'dT + I'di - I'd\pi_e + dG$
$L(Y, i) = M/P$	$L_Y dY + L_i di = dM/P - (M/P^2)dP$

Goods Market and Money Market; Labor Market has Fixed Wages (Short Run Equilibrium)

Exogenous (fixed)	Endogenous (4 unknowns)	Embedded Unknowns
G, M, π_e, T, K, W	Y, i, P, N	C, I

Equations (4)	Differentials
$F_N = W/P$	$F_{NN}dN + F_{NK}dK = dW/P - (W/P^2)dP$
$Y = F(K,N)$	$dY = F_K dK + F_N dN$
$Y = C(Y - T) + I(i - \pi_e) + G$	$dY = C'dY - C'dT + I'di - I'd\pi_e + dG$
$L(Y, i) = M/P$	$L_Y dY + L_i di = dM/P - (M/P^2)dP$

Open Economy - can be either long-run or short-run; add these 3 equations; add NX to $Y = C + I + G$

Exogenous (fixed)	Endogenous (3 more)
Y^*, P^*, i^*	e, E, NX

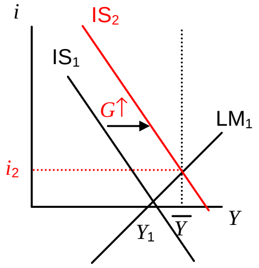
Equations (3 more)	Differentials
$NX = X(e, Y - T, Y^* - T^*)$	$dNX = X_e\, de + X_Y\, dY - X_Y\, dT + X_{Y^*}dY^* - X_{Y^*}dT^*$
$e = e(i - i^*)$	$de = e'di - e'di^*$
$e = EP^*/P$	$de = (P^*/P)dE + (E/P)dP^* - (EP^*/P^2)dP$

Note: to find multipliers for embedded unknowns, you need to go back to the functions embedded in the equations

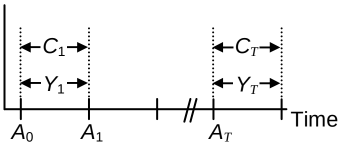
Embedded Equations	Differentials
$C = C(Y - T)$	$dC = C'dY - C'dT$
$I = I(i - \pi_e) + G$	$dI = I'di - I'd\pi_e$
$T = T(Y)$	$dT = T'dY$
	Plug into dC & get $dC = C'(1 - T')dY$

Consumption

- **$C(Y - T)$** - for IS-LM models in previous section, we looked at consumption as function of after-tax income only
- **$C(Y - T, i - \pi_e)$** - consumption as function of after-tax income and real interest rate
  - **$C_Y > 0$** - this was C' from before
  - **$C_r < 0$** - if interest rates increase, consumption decreases; from IS-LM: now $G\uparrow \Rightarrow I\downarrow$ & $C\downarrow$; there is "crowding out" of investment and consumption
- **Shape of Consumption Function** - Keynes argued for two things:
  - **$0 < C_Y < 1$** - spend more as $Y\uparrow$, but not as much as ΔY
  - **$APC = C/(Y - T)$** - average propensity to consume increases as $(Y - T)\downarrow$; same as saying average propensity to save decreases (i.e., rich people save more and consume less as proportion of income than poor people)
    - **Significance** - increasing output/worker (y) means increasing standard of living means we're getting richer as a society; should have $APC\downarrow$ (i.e., more savings)
    - **Problem** - $APC\downarrow$ ($APS\uparrow$) $\Rightarrow i\downarrow$; eventually get to lower limit on interest rate (zero); now economy can't get to potential by lowering wages
      - **Keynes' Solution** - $G\uparrow$ to fix economy
    - **Real World** - not evidenced in U.S., but could argue this happened to Japan ($i = 0$ from too much saving)
      - **Cross-Sectional Studies** - show that savings does go up as income goes up
      - **Time-Series Studies** - show that savings stay fairly constant (i.e., income doesn't affect savings rate)



- **Utility (U)** - firms are easy to explain; they seek profit; households are more difficult; we pretend we can quantify happiness or satisfaction (utility)
  - **$U(C,S)$** - utility as a function of consumption and savings; note: once again using poor notation by using U to represent both utility and the function for utility: $U = U(C,S)$
    - **Marginal Utility of Consumption** - $U' = \partial U/\partial C > 0$; if consumption increases, so does utility ("more is better")
  - **$U(C_1,C_2,...C_T)$** - people don't derive utility from savings; savings are used to finance future consumption; $\therefore$ look at utility as function of time series of consumption
    - **Future Consumption** - assumes people can plan for future consumption (i.e, rational & calculating); Bomberger: "There are 100,000 versions of irrational; that's too much work so we'll assume rational" (rough paraphrase)
- **Real Consumption** - accounts for price level
- **Saving (S)** - **$S_1 = Y_1 - C_1$**; labor income minus consumption; ignore other income for now because it's endogenous (depends on saving); all three (S, Y, C) are flows
- **Assets (A)** - stock of accumulated savings; **$A_1 = A_0(1 + r) + (Y_1 - C_1)(1 + r)$** ($r$ = real interest rate; assumes saving is done at beginning of year)



- **Time Horizon (T)** - last year for planning
  - **Bequest (A_T)** - assets remaining at time T; what you leave as your inheritance; must have **$A_T \geq 0$** or problem won't be constrained (always happier if you die in greater debt); but if $A_T \neq 0$ (i.e., $A_T > 0$), this becomes an intergenerational problem (hard to solve)
- **Intertemporal Budget Constraint** - have to choose consumption plan ($C_1,C_2,...,C_T$ & A_T) that you can afford; present value of spending and bequest must equal financial wealth plus present value of future labor; if you assume r is constant and saving is done at beginning of year: $A_2 = (Y_2 - C_2)(1 + r) + A_1(1 + r)$

Substitute A_1 from above so $A_2 = (Y_2 - C_2)(1 + r) + [(Y_1 - C_1)(1 + r) + A_0(1 + r)](1 + r) = (Y_2 - C_2)(1 + r) + (Y_1 - C_1)(1 + r)^2 + A_0(1 + r)^2$

Keep this up through the end of the time horizon and get:

$A_T = (Y_T - C_T)(1 + r) + (Y_{T\text{-}1} - C_{T\text{-}1})(1 + r)^2 + \ldots + (Y_2 - C_2)(1 + r)^{T\text{-}1} + (Y_1 - C_1)(1 + \mathrm{r})^T + A_0(1 + r)^T$

Put endogenous (choice) variables ($C_1, C_2, \ldots, C_T$ & A_T) on left side:

$$\underbrace{C_1 + \frac{C_2}{(1+r)} + \cdots + \frac{C_{T-1}}{(1+r)^{T-2}} + \frac{C_T}{(1+r)^{T-1}}}_{\text{Present Value of Spending}} + \underbrace{\frac{A_T}{(1+r)^{T-1}}}_{\substack{\text{PV of}\\ \text{Bequest}}} = \underset{\substack{\downarrow\\ \text{Financial}\\ \text{wealth}}}{A_0} + \underbrace{Y_1 + \frac{Y_2}{(1+r)} + \cdots + \frac{Y_{T-1}}{(1+r)^{T-2}} + \frac{Y_T}{(1+r)^{T-1}}}_{\substack{\text{Present Value of Future Labor}\\ \text{(human wealth/capital)}}}$$

Book Version - assumes interest added at end of year (not start); leaves out A_T, but since uses ≤ (rather than =), it assumes $A_T \geq 0$

$$\sum_{t=1}^{T} \frac{1}{(1+r)^t} C_t \leq A_0 + \sum_{t=1}^{T} \frac{1}{(1+r)^t} Y_t$$

No interest rate (r = 0): $\sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t$

Intertemporal Utility Function - **$U(C_1, C_2, \ldots C_T, A_T)$**

More is Better - $\partial U/\partial C_i$ = **$U_i > 0$**; usually also assume $\partial U/\partial A_T = U_A > 0$ (but could be < 0)

Diminishing Marginal Utility - $\partial^2 U/\partial C_i^2$ = **$U_{ii} < 0$**

Cross Terms - $\partial^2 U/\partial C_i \partial C_j$ could be anything; usually assume = 0; if ≠ 0, then consumption in one year leads to greater (or less) utility in some other year

Book Version - assumes $U_{ij} = 0$ ($i \neq j$); assumes utility function (u) is same form year to year

$$U = \sum_{t=1}^{T} u(C_i) = u(C_1) + u(C_2) + \ldots + u(C_T)\,;\ u'(\bullet) > 0,\ u''(\bullet) < 0$$

Analogy to Consumer Theory - Max $U(Q_1, Q_2, \ldots)$ subject to $P_1Q_1 + P_2Q_2 + \ldots = Y$; basically doing same thing except now we're doing it over time

Optimal Solution - consumption plan $C_1^*, C_2^*, \ldots, C_T^*$ that maximizes utility subject to budget

Example - (p345)

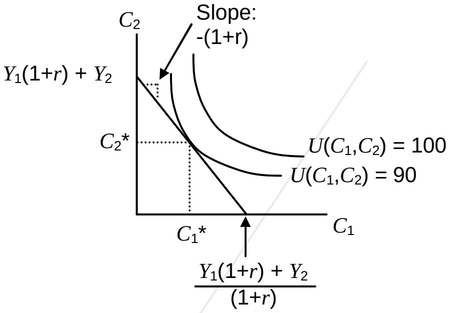
$$U = \sum_{t=1}^{T} \frac{1}{(1+\rho)^t} \frac{C_t^{1-\theta}}{1-\theta} = \frac{1}{1-\theta}\left[\frac{C_1^{1-\theta}}{1+\rho} + \frac{C_2^{1-\theta}}{(1+\rho)^2} + \cdots + \frac{C_{T-1}^{1-\theta}}{(1+\rho)^{T-1}} + \frac{C_T^{1-\theta}}{(1+\rho)^T}\right]$$

Risk Aversion (θ) - how severe are diminishing returns; determines willingness to shift consumption between different periods; smaller θ means marginal utility falls slower as consumption rises (more willing to allow consumption to vary over time); θ is coefficient of relative risk aversion (the inverse of the elasticity of substitution between consumption at different dates)

Discount Rate (ρ) - similar to interest rate in time value of money; ρ measures "time value of consumption"; consumption now is better then consumption later; larger ρ means more impatient (consumption now even better than later)

Check More is Better - $U_1 = \dfrac{C_1^{-\theta}}{1+\rho} > 0$

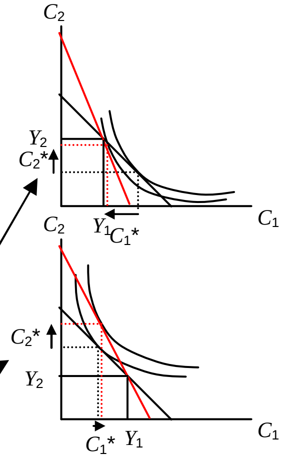
Check Diminishing MU - $U_{11} = -\dfrac{\theta C_1^{-\theta-1}}{1+\rho} < 0$



Simple Case - $T = 2$, $A_0 = 0$; $A_T = A_2 = 0$; $\partial U/\partial A_T = U_A = 0$

- Budget Constraint: $C_1 + \frac{C_2}{(1+r)} = Y_1 + \frac{Y_2}{(1+r)}$
- Solve constraint for C_2: $C_2 = Y_1(1+r) + Y_2 - C_1(1+r)$
  - Only 1 Choice: if you pick C_1, C_2 is automatically determined
- **Indifference Curve** - all combinations of C_1 and C_2 that result in same amount of utility
- **Optimal Solution** - indifference curve just tangent to budget line; from 1st order condition:
  - $U_1 - (1 + r)U_2 = 0 \Rightarrow$ you can't make yourself better off by saving more (or less)
  - **Interpretation** - extra dollar spent in C_1 gains U_1, but you lose $(1 + r)$ dollars in C_2 which is a loss of $(1 + r)U_2$
- **What Makes People Save** - take total differential of 1st order condition:
  - 1st Order Condition: $U_1(C_1) - (1 + r)U_2(Y_1(1 + r) + Y_2 - C_1(1 + r)) = 0$
  - Differential: $U_{11}dC_1 - U_2dr - (1 + r)U_{22}(Y_1dr + (1 + r)dY_1 + dY_2 - C_1dr - (1 + r)dC_1) = 0$
  - Combine terms & solve for dC1:
    - $U_{11}dC_1 - U_2dr - (1 + r)U_{22}[(Y_1 - C_1)dr + (1 + r)dY_1 + dY_2 - (1 + r)dC_1) = 0$
    - $U_{11}dC_1 + U_{22}(1 + r)^2dC_1 = U_{22}(1 + r)^2dY_1 + U_{22}(1 + r)dY_2 + (Y_1 - C_1)(1 + r)U_{22}dr + U_2dr$

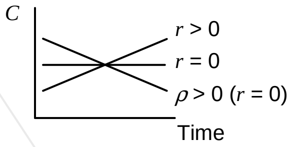
$$dC_1 = \boxed{\begin{array}{c}\frac{U_{22}(1+r)^2}{U_{11}+U_{22}(1+r)^2}\\ >0\end{array}}\,dY_1 + \boxed{\begin{array}{c}\frac{U_{22}(1+r)}{U_{11}+U_{22}(1+r)^2}\\ >0\end{array}}\,dY_2 + \boxed{\begin{array}{c}\frac{(Y_1-C_1)(1+r)U_{22}+U_2}{U_{11}+U_{22}(1+r)^2}\\ ??\end{array}}\,dr$$

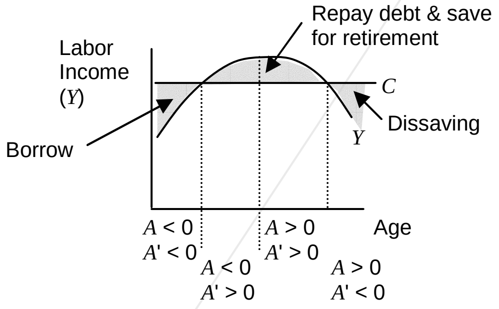


- **Interest Effect** - earning income now (Y_1) is better than next year (i.e., $dC_1/dY_1 > dC_1/dY_2$); evident in $(1 + r)^2$ term vs. $(1 + r)$; if $r = 0$, this effect goes away ($dC_1/dY_1 = dC_1/dY_2$)
- **Increase *r*** - budget line is steeper because consumption next year (C_2) is cheaper relative to this year; budget line rotates on point where interest is irrelevant (i.e., $C_1 = Y_1$ and $C_2 = Y_2$; consume exactly what you make; zero savings); effect on C_1 depends on savings in year 1 ($Y_1 - C_1$)
  - **Negative Savings (Borrowing)** - $Y_1 - C_1 < 0$; $dC_1/dr < 0$; makes sense because borrowing will be more expensive
  - **No Savings** - $Y_1 - C_1 = 0 \Rightarrow Y_2 = C_2$ (consuming at rotation point on graph); $dC_1/dr < 0$; make sense because you'll be better off saving
  - **Positive Savings** - $Y_1 - C_1 > 0$; dC_1/dr can be > 0 or < 0
    - **Income Effect** - more income available in year 2; can transfer some back to year 1; could have C_2↑ and C_1↑ with <u>positive savings</u>; negative savings will have C_1↓ (similar to consumer theory if P_{Beef}↓, buying same amount of beef, you now have more income to spend on other goods)
    - **Substitution Effect** - since consumption in year 2 is relatively cheaper than year 1, buy more of it (similar to consumer theory if P_{Beef}↓, substitute chicken with beef)

Interest (*r*) vs. Discount Rate (*ρ*) -

- $r = 0 \Rightarrow u_1 = u_2 = u_3 = \ldots$; ∴ with no discounting ($\rho = 0$), spend same amount every year
- $r > 0$ and $\rho > 0 \Rightarrow$ opposite effects; may cancel; people tend to "smooth out" consumption over time (borrow in lean years and save in good years); two models to explain this: life-cycle and permanent income





Life-Cycle Model

Basics - forward looking person can smooth out consumption; borrow early; repay debt and save during peak years; graph shows how assets (*A*) change over time (*A*')

Numerical Example -

	Young	Middle	Old	*C*
Poor	15K	25K	5K	15K
Middle	25K	35K	15K	25K
Rich	35K	45K	25K	35K

Assumptions - no uncertainty ; each period is same amount of time; no interest rate

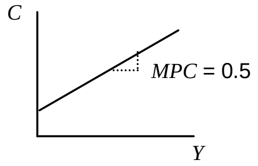
Consumption for $A_T = 0$ - i.e., *MPC* = 0; for poor person: (15K + 25K + 5K)/3 = $15K; for middle person: (25K + 35K + 15K)/3 = 25K; rich person: (35K + 45K + 25K)/3 = 35K

Budget Study - record income and consumption

Income	Consumption	Saving
5K	15K	-10K
15K	20K	-5K
25K	25K	0
35K	30K	5K
45K	35K	10K

(15K + 25K)/2
P M

(15K + 25K + 35K)/3
P M R



Problem - budget study makes it look like rich people (higher income) save more; plot above indicates *MPC* = 0.5, but model has no difference (all have *MPC* = 1)

Modigliani & Brumberg Model (1954) -

$$C_t = \frac{Y_t + (T-1)Y_t^e + A_t}{T}$$

Y_t^e = expected income on average from t + 1 to future; can't measure it so by assumption:

$$Y_t^e = \beta Y_t$$

Regression - used $C_t = a_1 Y_t + a_2 A_t$, where $a_1 = \frac{1+\beta(T-1)}{T}$ and $a_2 = \frac{1}{T}$

Result - $C_t = 0.7Y_t + 0.06A_t \Rightarrow \underbrace{\frac{C_t}{\text{GDP}}}_{MPC} = 0.7\underbrace{\frac{Y_t}{\text{GDP}}} + 0.06\underbrace{\frac{A_t}{\text{GDP}}}_{3} = 0.71$

Y_t = labor income
Y_t/GDP = proportion of GDP from wages = 0.75

Long-Run *MPC* = 0.71 > Short-Run *MPC* = 0.52

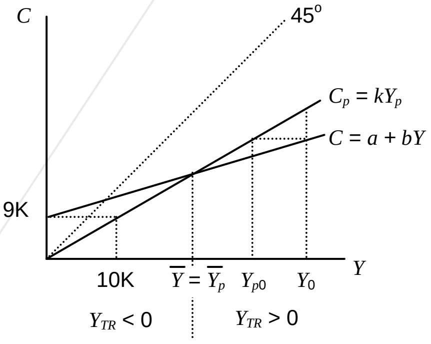
Interpretation of Short-Run - if GDP↓, each $1↓ ⇒ Y_t↓ $0.75 ∴ *C*↓ by 0.7(0.75) = $0.52; agrees with real world experience (consumption usually doesn't go down much during recessions)

Permanent Income Theory

Friedman Model (1957) - people plan with very long time horizon; indefinite because of heirs

$$\text{Wealth} = A_0 + \underbrace{Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2} + \cdots}_{\text{Human Capital (PV of labor income)}} \text{ (no end)}$$

Financial Capital



Permanent Income (Y_p) - sustainable level of consumption without diminishing wealth; $\boldsymbol{Y_p = r \cdot}$**wealth**

Transitory Income (Y_{TR}) - difference between permanent income and actual income; $\boldsymbol{Y = Y_p + Y_{TR}}$

Permanent & Transitory Consumption - $C = C_p + C_{TR}$

Model - $\boldsymbol{C_p = kY_p}$, k is factor of proportionality (MPC in long-run)

Assumptions -

1. $C_{TR} \sim C_p$ (deviations from average consumption not correlated)
2. $Y_{TR} \sim Y_p$ (same)
3. $k \sim Y_p$ (rich & poor have same MPC)
4. $C_{TR} \sim Y_{TR}$ (consumption varies from different circumstances than income

Numerical Example - using k = 0.9; no circumstances so $C_p = C$

	Yp	Bad	Avg	Good	C_p	C
Poor	10K	0	10K	20K	9K	9K
Middle	20K	10K	20K	30K	18K	18K
Rich	30K	20K	30K	40K	27K	27K

0.9(10K)

Budget Study -

Income (Y)	Consumption	MPC	Saving	Y_P	Y_{TR}	MPC
0	9K		-9K	10K	-10K	
10K	13.5K	0.45	-3.5K	15K	-5K	0.9
20K	18K	0.45	2K	20K	0	0.9
30K	22.5K	0.45	7.5K	25K	5K	0.9
40K	27K	0.45	13K	30K	10K	0.9

(9K + 18K + 27K)/3
P M R

$\Delta C/\Delta Y$ = (22.5K - 18K)/10K

(10K + 20K + 30K)/3
P M R

$\Delta C/\Delta Y_p$ = (22.5K - 18K)/5K

Problem - incorrect conclusion form single year's data ("rich save more than poor"); it's not just rich & poor, but people with good and bad years

Regression - $\boldsymbol{C_t = kY_{pt}}$

Problem - can't measure permanent income

Solution - use expectation weighted average of past income: $\boldsymbol{Y_{pt} = w_0Y_t + w_1Y_{t-1} + w_2Y_{t-2} + \ldots}$

Assumptions -

1. $\frac{w_1}{w_0} = \frac{w_2}{w_1} = \frac{w_3}{w_2} = \cdots = \lambda < 0$ (declining weights; recent history is more important)
2. $\sum_{i=1}^{\infty} w_i = 1$

Results - back before computers, Friedman ran regression for λ = 0.1, 0.2,..., 0.9; best result was λ = 0.7, k = 0.9; w_0 = 0.3, w_1 = 0.21, w_2 = 0.14,..., w_{30} = 0.01

Long-Run - $MPC = k = 0.9$

Short-Run - $Y_t\downarrow$ \$1 $\Rightarrow$ $Y_{pt}\downarrow$ by $w_0 = 0.3$ $\therefore C_t\downarrow$ by $w_0 \cdot k = 0.3(0.9) = 0.27$ ** consumption not affected that much by recessions

Robert Hall (1978) - paper found support for and against permanent income theory

Specific Utility Function - $U(C_1, C_2, \ldots) = \ln(C_1) + \frac{\ln(C_2)}{1+r} + \frac{\ln(C_3)}{(1+r)^2} + \cdots$

Test Assumptions -

$$U_1 = \frac{dU}{dC_1} = \frac{1}{C_1} > 0,\ U_{11} = \frac{d^2U}{dC_1^2} = \frac{-1}{C_1^2} < 0, \text{ and } U_{ij} = 0 \ (\forall\ i \neq j)$$

Consumption "Discount Rate" (ρ) - if $\rho > 0$, people are impatient; rather consume more now rather than later

First Order Conditions - $U_1 = (1 + r)U_2$; U_1 is what you lose for reducing consumption in year 1; $(1 + r)U_2$ is what you gain in year 2 for reducing consumption in year 1

General Case - $U_t = (1 + r)U_{t+1}$

Year 1 - $\frac{1}{C_1} = \frac{1+r}{(1+\rho)C_2}$ ... solve for C_2: $C_2 = \frac{1+r}{1+\rho}C_1$

So $r\uparrow \Rightarrow$ move consumption to year 2; $\rho\uparrow \Rightarrow$ move consumption to year 1

Year 2 - $\frac{1}{(1+\rho)C_2} = \frac{1+r}{(1+\rho)^2 C_3}$ ... solve for C_3: $C_3 = \frac{1+r}{1+\rho}C_2 = \left(\frac{1+r}{1+\rho}\right)^2 C_1$

General Case - $C_t = \left(\frac{1+r}{1+\rho}\right)^{t-1} C_1$

Substitute 1st Order conditions into intertemporal budget constraint:

$$C_1 + \frac{\left(\frac{1+r}{1+\rho}\right)C_1}{(1+r)} + \frac{\left(\frac{1+r}{1+\rho}\right)^2 C_1}{(1+r)^2} + \cdots = A_0 + Y_1 + \frac{Y_2}{(1+r)} + \frac{Y_3}{(1+r)^2} + \cdots$$

$$C_1\left(1 + \frac{1}{(1+\rho)} + \frac{1}{(1+\rho)^2} + \cdots\right) = A_0 + Y_1 + \frac{Y_2}{(1+r)} + \frac{Y_3}{(1+r)^2} + \cdots$$

Solve for C_1:

$$C_1 = \frac{\rho}{1+\rho}\left(A_0 + Y_1 + \frac{Y_2}{(1+r)} + \frac{Y_3}{(1+r)^2} + \cdots\right)$$

Problem - still can't measure future incomes (Y_2, Y_3,...)

Expected Future Income $E_i(Y_j)$ - expectation in year i of income in year j

Solution -

$$C_1 = \frac{\rho}{1+\rho}\left(A_0 + Y_1 + \frac{E_1(Y_2)}{(1+r)} + \frac{E_1(Y_3)}{(1+r)^2} + \cdots\right)$$

Year 2:

$$C_2 = \frac{\rho}{1+\rho}\left(A_1 + Y_2 + \frac{E_2(Y_3)}{(1+r)} + \frac{E_2(Y_4)}{(1+r)^2} + \cdots\right)$$

Rational Expectation - will look at optimal forecast; $E_1(Y_3)$ contains all pertinent information in year 1 regarding income in year 3; $E_2(Y_3)$ updates this information so expected value could be different, but the difference should only result from new information (i.e., no lagged variables should be significant)

Know Now or Later - some use Y_1, others use $E_1(Y_1)$, depends on assumption of when information becomes available; not critical for results

Assume $r = 0$ (for simplicity) and consume same amount each period (permanent income theory), now:

$$C_1 = \left[A_0 + E_1(Y_1) + E_1(Y_2) + E_1(Y_3) + \cdots\right]/T$$

$$C_2 = \left[A_1 + E_2(Y_2) + E_2(Y_3) + E_2(Y_4) + \cdots\right]/(T-1)$$

Substitute $A_1 = A_0 + Y_1 - C_1$ and add terms like $E_1(Y_i) - E_1(Y_i) = 0$ (i = 2, 3,...)

$$C_2 = \left[A_0 + Y_1 - C_1 + E_2(Y_2) + E_2(Y_3) + \cdots + [E_1(Y_2) - E_1(Y_2)] + [E_1(Y_3) - E_1(Y_3)] + \cdots\right]/(T-1)$$

Swap the $E_2(Y_i)$ and $E_1(Y_i)$ terms

$$C_2 = \left[A_0 + Y_1 - C_1 + E_1(Y_2) + E_1(Y_3) + \cdots + [E_2(Y_2) - E_1(Y_2)] + [E_2(Y_3) - E_1(Y_3)] + \cdots\right]/(T-1)$$

Forecast Revision - $E_2(Y_i)$ - $E_1(Y_i)$

Add $E_1(Y_1)$ - $E_1(Y_1)$

$$C_2 = \left[A_0 + Y_1 - C_1 + E_1(Y_1) + E_1(Y_2) + \cdots + E_1(Y_1) + [E_2(Y_2) - E_1(Y_2)] + \cdots\right]/(T-1)$$

Rearrange terms and substitute $E_2(Y_1) = Y_1$ (perfect info after the fact so year 2's expected value of income in year one is the actual income from year 1)

$$C_2 = \left[-C_1 + \underbrace{A_0 + E_1(Y_1) + E_1(Y_2) + \cdots}_{T\cdot C_1} + \underbrace{[E_2(Y_1) - E_1(Y_1)] + [E_2(Y_2) - E_1(Y_2)] + \cdots}_{\text{Revision on all forecasts}}\right]/(T-1)$$

Plug in $T\cdot C_1$ and collect terms

$$C_2 = \left[C_1(T-1) + [E_2(Y_1) - E_1(Y_1)] + [E_2(Y_2) - E_1(Y_2)] + \cdots\right]/(T-1)$$

Break up sum to cancel (T - 1) in first term

$$C_2 = C_1 + \left[[E_2(Y_1) - E_1(Y_1)] + [E_2(Y_2) - E_1(Y_2)] + \cdots\right]/(T-1)$$

Finding - only change consumption if there's been some change in expected future income (i.e., $C_2 = C_1$ unless forecasts change)

What Changes Forecasts - from rational expectations (& math shown above), only new information changes forecasts (hence consumption); any lagged terms should be insignificant: $Ct = C_{t-1} + aY_{t-2}$ should yield $a = 0$ because C_{t-1} already incorporates Y_{t-2}

Regression - used quarterly consumption data (1948:1 to 1977:1) for services and non-durables

Consumption Categories - national income and product accounts include 3 types of consumption: durables, non-durables, and services

No Durables - Hall left out durables because he argued they are more like savings or investment; purchase is done at one time, but consumption is taken over a period of time; Example: car provides transportation service; NIPA records sale in year 1 as consumption, but services consumed last longer (10 years in Bomberger's case); so consumption of service is smooth over time, but purchases aren't

Confirmation - $C_t = 1.02C_{t-1} - 0.01Y_{t-1}$ (model used Y_1 not $E(Y_1)$)
(0.04) (0.03) $R^2 = 0.9988$

Confirmation of theory because coefficient for Y_{t-1} is insignificant

Discredit - another regression using inputs that shouldn't affect C_t discredits the theory

$C_t = 1.01C_{t-1} + 0.223S_{t-1} - 0.258S_{t-2} + 1.67S_{t-3} - 0.120S_{t-4}$ (S = index of stock prices)
(0.01) (0.05) (0.08) (0.08) (0.05)

Evidence against theory; shouldn't have lagged variables beyond t - 1 that are significant in determining C_t; that information should be captured in C_{t-1}

John Shea (1995) - many tried to find other ways to discredit permanent income theory and rational expectations; Shea focused on fact that Hall used aggregate income so he didn't catch individuals with lower income resulting from job loss or illness; Shea used survey of income dynamics that followed individual households; focused on those with union labor because of the predictable income; 647 observations

Regression - $C_t = C_{t\text{-}1} + aY_{t\text{-}1}$; $Y_{t\text{-}1}$ is predictable component of income taken from labor contracts; a = 0.89 (0.46), large impact, but not statistically significant

Salvaging Personal Income Theory - economists like the theory so there are several explanations for why empirical tests seem to contradict it

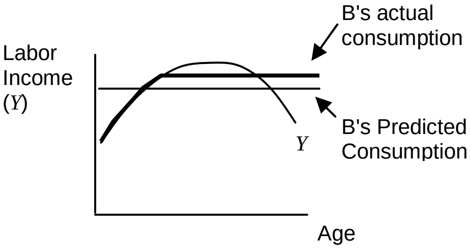
Liquidity Constraints - borrowing to maintain constant consumption may not be possible because banks don't care about person's expected future income; difficult to borrow against human capital

Person A - A_0 large and $E_1(Y_{10})$ low; will save a lot now for future consumption

Person B - A_0 small and $E_1(Y_{10})$ large; would like to borrow now, but banks won't let him

Person C - A_0 large (inheritance) and $E_1(Y_{10})$ large; will run down assets (equivalent of borrowing)

Predictions - life-cycle model and permanent income theory predict person A and C; it's not possible for person B to behave like C which is what theory says; instead B will have lower consumption in early years, but will make up for it with higher consumption in future then he would've otherwise because he won't have to repay debt (see graph)



Savings

Misconception - many people say U.S. doesn't save much, but our GDP growth is same or higher than other industrialized countries over last 10-15 years; data from 1990s as % GDP:

Savings	U.S.	Germany	UK	Canada
National	16.5	21.9	15.7	17.5
Government	-2.9	-2.6	-3.7	-4.4
Private	19.4	24.5	19.4	21.9

What Savings - data above show U.S. saving isn't much different, but reports on low U.S. savings tend to focus on personal savings

2002 Data - savings for U.S. as % of GDP

Business Savings - U.S. corporations hardly pay dividends (because of tax laws); instead they have retained earnings (saving); people own corporations and account for business savings in their personal savings

National	15.1
Government	-0.2
Business	12.5
Personal	2.8

Government Policy

Budget Constraint - if people plan and are forward looking, they plan budget based on after-tax income

$$C_1 + \frac{C_2}{(1+r)} + \frac{C_3}{(1+r)^2} + \cdots = A_0 + (Y_1 - T_1) + \frac{Y_2 - T_2}{(1+r)} + \frac{Y_3 - T_3}{(1+r)^2} + \cdots$$

Expected Future Taxes ($E_1(T_i)$) - taxes people expect to pay in year i based on information in year 1

$$C_1 + \frac{C_2}{(1+r)} + \frac{C_3}{(1+r)^2} + \cdots = A_0 + (Y_1 - T_1) + \frac{E_1(Y_2) - E_1(T_2)}{(1+r)} + \frac{E_1(Y_3) - E_1(T_3)}{(1+r)^2} + \cdots$$

- **Rebate vs. Rate Cut** - tax rebate reduces T_1 (or some other specific year); tax rate cut changes all future T_i; ∴rate cuts which haven't occurred could still influence current consumption based on life-cycle model and permanent income theory (people smooth consumption)

Government Budget - $B_1 = B_0(1+r) + G_1 - T_1$

- **Budget Surplus/Deficit** - revenue minus interest expense minus government purchases (T_1 - rB_0 + G_1); positive value is a surplus; negative value is a deficit
- **Net Taxes (T_i)** - taxes collected minus transfer payments in year i
  - **Note:** T↑ means tax revenue increase; says nothing about tax rates; if lowering tax rate increases revenue you still have T↑
- **Government Debt (B_i)** - bonds outstanding in year i; can rewrite equation above to show debt from previous year minus surplus (or plus deficit): $B_1 = B_0 + rB_0 + G_1 - T_1$

Use same formula for B_2:

$$B_2 = B_1(1+r) + G_2 - T_2$$

Substitute B_1:

$$B_2 = B_0(1+r)^2 + (G_1 - T_1)(1+r) + G_2 - T_2$$

Look at general case:

$$B_N = B_0(1+r)^N + (G_1 - T_1)(1+r)^{N-1} + (G_2 - T_2)(1+r)^{N-2} + \cdots G_N - T_N$$

- **Government Budget Constraint** - put G_1, G_2, ..., G_N & B_0 on left side (similar to intertemporal budget constraint):

$$\underbrace{G_1 + \frac{G_2}{1+r} + \frac{G_3}{(1+r)^2} + \cdots + \frac{G_N}{(1+r)^{N-1}}}_{\text{Present Value of Purchases}} + \underset{\substack{\downarrow \\ \text{Current} \\ \text{Debt}}}{B_0} = \underbrace{T_1 + \frac{T_2}{1+r} + \frac{T_3}{(1+r)^2} + \cdots + \frac{T_N}{(1+r)^{N-1}}}_{\text{Present Value of Future Income}} + \underbrace{\frac{B_N}{(1+r)^{N-1}}}_{\substack{\text{PV of Future} \\ \text{Debt}}}$$

Substitute this into personal intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2} + \cdots = A_0 - B_0 + (Y_1 - G_1) + \frac{Y_2 - G_2}{1+r} + \frac{Y_3 - G_3}{(1+r)^2}$$

- **Interpretation** - people view government spending as a liability to them; ∴ C is unchanged when T↓ with G fixed because people view it as an increase in liability in future (i.e., if G doesn't change, T will have to be raised again in future)
  - **Previous Models** - IS-LM multipliers and permanent income theory suggested that T↓ with fixed G would increase consumption
- **Ricardian Equivalence** - tax finance and debt finance are equivalent; cutting taxes with fixed G doesn't change consumption because people save ΔT in anticipation of having to pay it back in the future; $C(T - Y)$ assumption is incorrect
- **Criticism** - there's no logical flaw with Ricardian Equivalence, but people argue the assumptions: rather than being forward looking, many economists say people are short-sighted; also some don't like the infinite horizon of the permanent income model; this second criticism was addressed by Barro

Overlapping Generations Model (Barro 1974) - Assume each generation lives 2 periods and overlaps the next generation by 1 period; each has a utility function based on their consumption in those two periods AND the utility of their offspring

Generation A: $U^A(C_1^A, C_2^A, U^B)$, Generation B: $U^B(C_1^B, C_2^B, U^C)$, etc.

Assumptions - $U_1^A = \dfrac{\partial U^A}{\partial C_1^A} > 0$, $U_2^A = \dfrac{\partial U^A}{\partial C_2^A} > 0$, $U_3^A = \dfrac{\partial U^A}{\partial U^B}$ depends

Hate Kids - leave debt; $U_3^A < 0$; not allowed by assumption

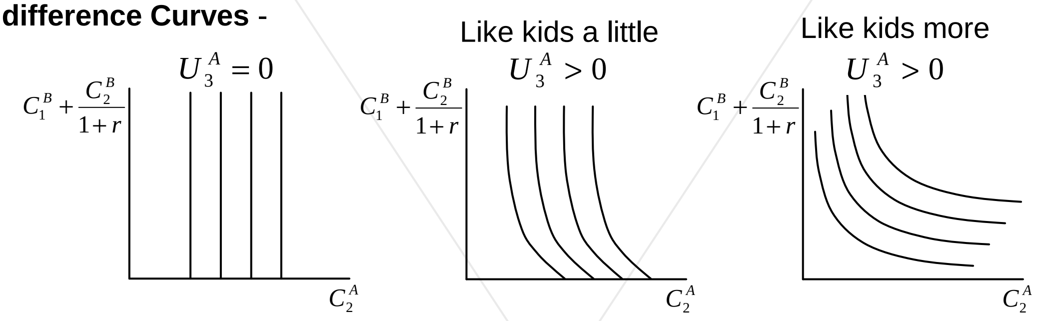
No Bequest - don't leave anything for kids; $U_3^A = 0$

Like Kids - leave something for kids; doesn't have to be inheritance, could be transfer during their lives (e.g., school, car, etc.); $U_3^A > 0$

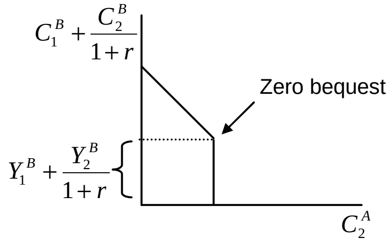
1st Order for Consumption - $U_1^A = (1+r)U_2^A$

1st Order for Bequest - if $U_3^A > 0$ (i.e., leaving bequest), $U_2^A = U_1^B U_3^A$ (i.e., marginal utility for consumption in period 2 for generation A equals increased utility of consumption for generation B times how this increase in utility for B increases utility for A)

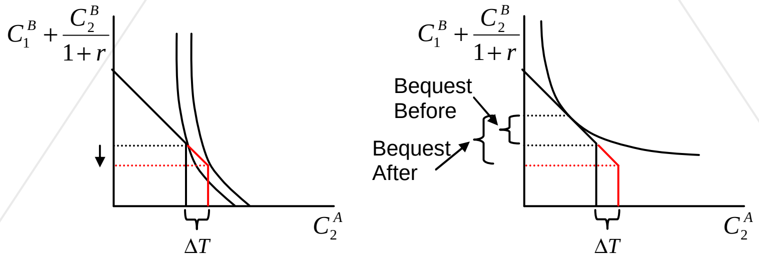
Indifference Curves -



Budget Constraint - slope of -1; transfer \$1 from generation A means generation B gets \$1 because transfer occurs in same time period; vertical intercept is sum of total income for both generations; interpretation here is questionable since generation A can't possibly transfer 100% of income to generation B; since generation A can't leave debt for generation B (by assumption), budget line stops before reaching horizontal axis



Tax Cut - if $T\downarrow$ for generation A with no change in G, two cases: (a) generation A spends all of it, they reduce consumption for generation B (because generation B will eventually be faced with $T\uparrow$ to pay for tax cut); (b) benefits of tax cut get passed to generation B; technically there's a third cases where the tax cut gets split, but it requires a funny shaped indifference curve



Implication - $T\downarrow$ doesn't change consumption (just like Recardian Equivalence suggests, but now we see it with finite lifetimes); if you don't believe in Recardian Equivalence because of finite lifetimes, then people wouldn't leave bequests, but in the real world they do; finite lifetime argument against Recardian Equivalence is weak

Time Series Models

Looking at how expectations are formed; two choices:

Root Causes - look at root causes (e.g., what's the Fed doing with interest rates?)

Time Series - look at past, usually place more emphasis on recent events, and extrapolate from past patterns

Stat Review

Sample Mean - $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

Population Mean - $\mu = E(X)$

Sample Variance - $s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$

Population Variance - $\sigma^2 = E\left[(X-\mu)^2\right]$

Covariance - measures linear relationship between two variables (e.g., X↑ and Y↑ at same time is a positive relationship); $E\left[(X-\mu_X)(Y-\mu_Y)\right] = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})$

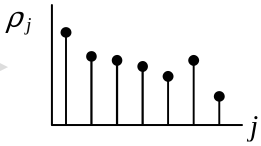
Autocovariance *j* - measures linear relationship between variable and itself lagged *j* periods;

$$\frac{1}{n-j}\sum_{i=1}^{n-j}(X_{j+i} - \overline{X})(X_i - \overline{X})$$

Autocorrelation ρ_j - normalizes autocovariance; $\rho_j =$ autocovariance *j* / variance

$\rho_0 = 1$ (by definition); larger ρ_j means stronger relationship to lagged value

Correlogram - graph of ρ_j versus *j*



Models

White Noise - random error term, u_t, that satisfies four assumptions:

$E(u_t) = 0$ - on average error is zero; positive errors balance out negative errors

$Var(u_t) = E(u_t^2) = \sigma_u^2$ - error terms have constant variance

Note: smaller σ_u^2 means model is a **better fit** to the data

$E(u_t u_s) = 0$ (for $t \neq s$) - error terms aren't related to each other

$\rho_j = 0$ (for *j* = 1,2,...,*n*) - follows from previous assumption

Model - want to find a model that gives an error term that is just white noise

Simplest Model - just use the mean and say all variations are "white noise"

Example - $\pi_t = \overline{\pi} + u_t$

Streaks - value variable takes one depends on recent values (e.g., if previous value is negative, it's more likely for next value to be negative); simple model doesn't work well in this case

ARMA Models

First Order Moving Average, MA(1) - assumes information from previous period is still important (i.e., $Cov(u_t,u_{t-1}) > 0$)

Model - $\boxed{\pi_t = \mu + u_t + \alpha u_{t-1}}$

$$Var(\pi_t) = E\left[(\pi_t - \mu)^2\right] = E\left[(\mu + u_t + \alpha u_{t-1} - \mu)^2\right] = E(u_t^2 + 2\alpha u_t u_{t-1} + \alpha^2 u_{t-1}^2) =$$

$$E(u_t^2) + \alpha^2 E(u_{t-1}^2) = \sigma_u^2 + \alpha^2 \sigma_u^2 = (1+\alpha^2)\sigma_u^2$$

Note: $E(u_t u_{t-1}) = 0$ by assumption that u_t is white noise

$$Cov(\pi_t, \pi_{t-1}) = E\left[(\pi_t - \mu)(\pi_{t-1} - \mu)\right] = E\left[(\mu + u_t + \alpha u_{t-1} - \mu)(\mu + u_{t-1} + \alpha u_{t-2} - \mu)\right] =$$

$$E(u_t u_{t-1} + \alpha u_t u_{t-2} + \alpha u_{t-1}^2 + \alpha^2 u_{t-1} u_{t-2}) = \alpha E(u_{t-1}^2) = \alpha \sigma_u^2$$

Note: *Cov* of other lagged terms = 0 (by assumption since model only lags 1 period)

Autocorrelations - $\rho_0 = 1$, $\rho_1 = \alpha/(1-\alpha)$, $\rho_2 = \rho_3 = \cdots = 0$

***q*th Order Moving Average, MA(*q*)** - assumes information from previous q periods is still important

Model - $\boxed{\pi_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \cdots + \alpha_q u_{t-q}}$

$$Var(\pi_t) = (1 + \alpha_1^2 + \alpha_2^2 + \cdots + + \alpha_q^2)\sigma_u^2$$

$$Cov(\pi_t, \pi_{t-i}) = \begin{cases} (\alpha_i + \alpha_1 \alpha_{1+i} + \alpha_2 \alpha_{2+i} + \cdots + \alpha_{q-i}\alpha_q)\sigma_u^2 & i = 1,2,\ldots,q \\ 0 & i > q \end{cases}$$

First Order Autoregressive Process, AR(1) - more economical way of having longer lags; equivalent to MA(∞) with exponentially decreasing coefficients; model regresses this year's error (e_t) on last year's (e_{t-1})

Model - $\boxed{\pi_t = \mu + e_t, \text{ where } e_t = u_t + \beta e_{t-1}}$ (as before u_t is white noise)

Note: error terms are embedded:

$$\pi_t = \mu + e_t = \mu + u_t + \beta e_{t-1} = \mu + u_t + \beta u_{t-1} + \beta^2 e_{t-2} = \mu + \sum_{i=0}^{\infty} \beta^i u_{t-i}$$

$$Var(\pi_t) = \sigma_u^2 \sum_{i=0}^{\infty} \beta^i = \frac{\sigma_u^2}{1-\beta^2}$$

Autocorrelations - $\rho_i = \beta^i$

Coefficient - $\beta < 1$; determines weight of previous periods; larger value puts more emphasis on past; $\beta = 0$ is same as basic model (mean); $\beta = 1$ weights all periods equally

***p*th Order Autoregressive Process, AR(*p*)** -

Model - $\boxed{\pi_t = \mu + e_t, \text{ where } e_t = u_t + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \cdots + \beta_p e_{t-p}}$

ARMA(*p*,*q*) - combination on AR(p) and MA(q)

Model - $\boxed{\pi_t = \mu + e_t,\ e_t = u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \cdots + \alpha_q u_{t-q} + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \cdots + \beta_p e_{t-p}}$

Summary -

Simple Model - just using the mean; +: unbiased, easy

ARMA - +: better forecast for next period (accounts for streaks)

Forecasting

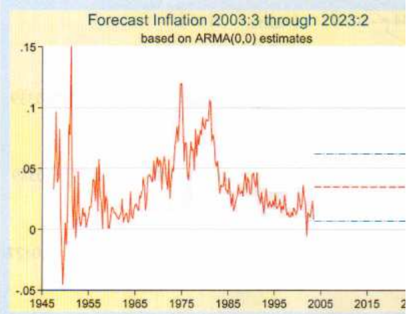
Forecasting - assuming people optimize so their expectations coincide with econometric forecasts; for this class just use the point estimate ± one standard deviation (we're ignoring the standard errors of the parameters and assuming the process doesn't change)

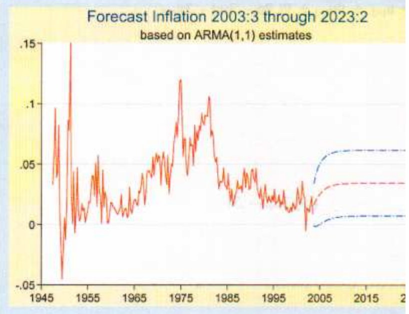
Point Estimate - conditional mean of what you want to forecast

Conditional Mean - $E_t(X)$; based on info available at time t

Forecast Standard Deviation - $E\left[(\pi_{t+i} - E_t(\pi_{t+i}))^2\right]$

Error (e_t) - difference between actual value and mean; $e_t = \pi_t - \mu$





ARMA(0,0) - same as white noise model; all predictions will be the same

$\pi_t = \mu + u_t \Rightarrow$ (inflation data) $\Rightarrow \pi_t = 0.0348 + u_t$ with $\sigma_u = 0.0274$

(0.0021)

Point Estimate - $E_{t+1}(\mu + u_{t+1}) = \mu$

StDev - $E\left[(\pi_{t+1} - E_t(\pi_{t+1}))^2\right] = E\left[(\mu + u_t - E_t(\mu + u_t))^2\right] = E\left(u_t^2\right) = \sigma_u^2$

(remember that $E(u_t) = E_t(u_t) = 0$)

Confidence Interval - 0.0348 ± 0.0274

ARMA(1,1) - will have biased forecast (different than long-run average), but a smaller standard deviation; standard deviation increases as the period you forecast gets further away; eventually gets back to same forecast as ARMA(0,0)

$\pi_t = \mu + u_t + \beta e_{t-1} + \alpha u_{t-1} \Rightarrow \pi_t = 0.0344 + u_t + 0.875 e_{t-1} - 0.302 u_{t-1}$ with $\sigma_u = 0.0176$

(0.0074) (0.038) (0.047)

Point Estimates - **Note:** at time t, e_t and u_t are known values; $E_t(u_{t+i}) = 0$ (i = 1,2,...)

$$E_t(\pi_{t+1}) = E(\mu + u_{t+1} + \beta e_t + \alpha u_t) = \boxed{\mu + \beta e_t + \alpha u_t} \rightarrow E_t(e_{t+1}) \quad E_t(e_{t+2}) \leftarrow$$

$$E_t(\pi_{t+2}) = E(\mu + u_{t+2} + \beta e_{t+1} + \alpha u_{t+1}) = \mu + \beta E_t(e_{t+1}) + \alpha E_t(u_{t+1}) = \boxed{\mu + \beta(\beta e_t + \alpha u_t)}$$

$$E_t(\pi_{t+3}) = E(\mu + u_{t+3} + \beta e_{t+2} + \alpha u_{t+2}) = \mu + \beta E_t(e_{t+2}) + \alpha E_t(u_{t+2}) = \boxed{\mu + \beta(\beta(\beta e_t + \alpha u_t))}$$

Note: since $\beta < 1$, $\lim_{i \to \infty} E_t(\pi_{t+i}) = \mu$

Variances - (StDev)^2

$$E_t\left[(\pi_{t+1} - E_t(\pi_{t+1}))^2\right] = E_t\left[((\mu + u_{t+1} + \beta e_t + \alpha u_t) - (\mu + \beta e_t + \alpha u_t))^2\right] = \boxed{E\left(u_{t+1}^2\right) = \sigma_u^2}$$

$$E_t\left[(\pi_{t+2} - E_t(\pi_{t+2}))^2\right] = E_t\left[((\mu + u_{t+2} + \beta e_{t+1} + \alpha u_{t+1}) - (\mu + \beta^2 e_t + \alpha\beta u_t))^2\right] =$$

$$E_t\left[(\mu + u_{t+2} + \beta(u_{t+1} + \beta e_t + \alpha u_t) + \alpha u_{t+1} - \mu - \beta^2 e_t - \alpha\beta u_t)^2\right] =$$

$$E_t\left[(u_{t+2} + (\alpha + \beta) u_{t+1})^2\right] = E_t\left[(u_{t+2}^2 + (\alpha + \beta)^2 u_{t+1}^2)\right] = \boxed{\left(1 + (\alpha + \beta)^2\right)\sigma_u^2}$$

Tricks - $e_{t+1} = u_{t+1} + \beta e_t + \alpha u_t$; $E(u_t u_s) = 0$ (for $t \neq s$) so you can ignore cross terms

$$E_t\left[(\pi_{t+3} - E_t(\pi_{t+3}))^2\right] = \boxed{\left(1 + (1 + \beta^2)(\alpha + \beta)^2\right)\sigma_u^2}$$ (algebra omitted)

Confidence Intervals - first need to find u_t and e_t... to get u_t you have to go back all the way to $t = 0$ and assume $u_0 = 0$ and $e_0 = 0$, then $u_1 = \pi_1 - \mu$ and you work your way up to u_t (or just have the computer do it for you); in this case $u_{2003:2} = -0.0148$

Last period of data: $\pi_{2003:2} = 0.0086 \therefore e_{2003:2} = 0.0086 - 0.0344 = -0.0258$

2003:3 (i.e., $t + 1$) -

$E(\pi_{2003:3}) = 0.0344 + 0.875(-0.0258) - 0.302(-0.0148) =$ **0.0163**

StDev = σ_u = **0.0176**

Confidence Interval: 0.0163 ± 0.0176

2003:4 (i.e., $t + 2$) -

$E(\pi_{2003:4}) = 0.0344 + 0.875(0.875(-0.0258) - 0.302(-0.0148)) =$ **0.0186**

StDev = $[(1 + (-0.302 + 0.875)^2)]^{1/2}\, 0.0176$ = **0.0203**

Confidence Interval: 0.0186 ± 0.0233

2004:1 (i.e., $t + 3$) -

$E(\pi_{2004:1}) = 0.0344 + 0.875(0.875(0.875(-0.0258) - 0.302(-0.0148))) =$ **0.0205**

StDev = $[(1 + (1 + 0.875^2)(-0.302 + 0.875)^2)]^{1/2}\, 0.0176$ = **0.0221**

Confidence Interval: 0.0205 ± 0.0221

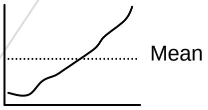
ARIMA

Unconditional Forecast - just use mean and standard deviation

Conditional Forecast - uses other information; gives tighter prediction in first few periods, then becomes same as unconditional forecast in later periods (e.g., ARMA)

Stationary - data is stationary if unconditional moments (i.e., mean and variance) exist and are constant

Example of Non-Stationary - GDP deflator:



Inflation & Price Level - $\pi_t = P_t - P_{t-1}$

Inflation is Stationary - shown before that ARMA works for forecasting π

Price Level Isn't - forecasting price level is first difference

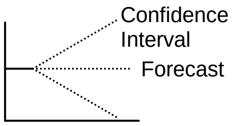
First Difference - example: $P_t = \pi_t + P_{t-1}$; variable lagged on itself

ARIMA(p,d,q) - d term is number of times you difference variable in question (or integrate the original variable);

Example - above we're differencing price level or integrating P_{t-1}; either way it's first difference

Formally - $X \sim$ ARIMA(p,1,q) if $x_t \equiv X_t - X_{t-1} \sim$ ARMA(p,q)

Random Walk - equally likely to go up or down; occurs when $\pi_t \sim$ ARIMA(0,0,0) (white noise) and $P \sim$ ARIMA(0,1,0); simplifies to $P_t = u_t + u_{t-1} + u_{t-2} + \ldots$; random walk ($P$) is not stationary, but it is "difference" stationary (i.e., π is stationary)



Forecast Revisions

Revision - if $\pi_{t+1} \neq E_t(\pi_{t+1})$, we're pretty sure $E_t(\pi_{t+2})$ won't be right so we need to update (revise) the forecast by figuring out $E_{t+1}(\pi_{t+2})$

Theory

1) Find $e_{t+1} = \pi_{t+1} - \mu$ (don't really need this for theoretical approach)

2) Find $u_{t+1} = \pi_{t+1} - E_t(\pi_{t+1})$

3) Revision (for t+2 only):

$\pi_{t+2} = \mu + u_{t+2} + \beta e_{t+1} + \alpha u_{t+1} \Rightarrow$ (direct way) $E_{t+1}(\pi_{t+2}) = \mu + \beta e_{t+1} + \alpha u_{t+1}$

Substitute $e_{t+1} = u_{t+1} + \beta e_t + \alpha u_t$

$$\pi_{t+2} = \mu + u_{t+2} + \beta(u_{t+1} + \beta e_t + \alpha u_t) + \alpha u_{t+1} = \mu + u_{t+2} + \beta(\beta e_t + \alpha u_t) + (\alpha + \beta)u_{t+1}$$

$$E_{t+1}(\pi_{t+2}) = \mu + \beta(\beta e_t + \alpha u_t) + (\alpha + \beta)u_{t+1} = \boxed{E_t(\pi_{t+2}) + (\alpha + \beta)u_{t+1}}$$

Therefore, forecast only changes if the earlier forecast was wrong (i.e., $u_{t+1} \neq 0$)

General Result - for ARMA(p,q):

$E_{t+1}(\pi_{t+2}) - E_t(\pi_{t+2}) = \lambda u_{t+1}$ where $\lambda = f(\alpha_1, \alpha_2, \ldots, \alpha_q, \beta_1, \beta_2, \ldots, \beta_p)$

$u_{t+1} = 0 \Rightarrow E_{t+1}(\bullet) = E_t(\bullet)$ (i.e., if no error in original forecast, future forecasts don't change)

Example - using data from above: $E_{2003:2}(\pi_{2003:3}) = 0.0163$; what if $\pi_{2003:3} = 0.0250$

Step 1 - $e_{2003:3} = \pi_{2003:3} - \mu = 0.0250 - 0.0344 =$ **-0.0094**

Step 2 - $u_{2003:3} = \pi_{2003:3} - E_{2003:2}(\pi_{2003:3}) = 0.0250 - 0.0163 =$ **0.0087**

Step 3 - $E_{2003:3}(\pi_{2003:4}) = E_{2003:2}(\pi_{2003:4}) + (\alpha + \beta)u_{2003:3} =$
0.0186 + (0.875 - 0.302)0.0087 = **0.0236**

Direct way: $E_{2003:3}(\pi_{2003:4}) = \mu + \beta e_{2003:3} + \alpha u_{2003:3} =$
0.0344 + 0.875(-0.0094) - 0.302(0.0087) = 0.0235 (rounding error)

Makes Sense - since original estimate was too low, the new estimate will be greater than the old one (0.0236 vs. 0.0186)

Permanent Income Theory

Look at impact of forecast revisions on permanent income theory (ignoring interest rates)

$$C_t = k\left(E_t(Y_{t+1}) + E_t(Y_{t+2}) + E_t(Y_{t+3}) + \cdots\right)$$

At the end of year t, we can calculate ut and revise forecasts:

$$C_{t+1} = k\left(E_{t+1}(Y_{t+2}) + E_{t+1}(Y_{t+3}) + E_{t+1}(Y_{t+4}) + \cdots\right)$$

Cases:

u_t	C_{t+1} (vs. C_t)
> 0	↑
= 0	no change
< 0	↓

∴ Consumption is a random walk!

Comments on Rational Expectations & HW4 - change sin consumption shouldn't be related to old news (i.e., no lagged variables are significant)

$Y_t = 1000 + u_t$ - $Y_t = 1200$ shouldn’t matter; next period still expect $Y_{t+1} = 1000$; rational person shouldn't change consumption in this case

$Y_t = Y_{t-1} + u_t$ - now all change sin income are permanent; if $u_t = 10$, then expect all future income to be higher by 10; consumption should change

$Y_t = Y_{t-1} + e_t$ where $e_t = u_t + 0.5e_{t-1}$ - $U_t = 10$ means 10 is permanent increase, but still have increasing in future; levels off with $\Delta Y = 20$ so consumption should increase by MPC(20) = .9(20) = 18

Investment

Investment - looked at it before as a function of real interest rate: $I = I(i - \pi^e)$, were $I' < 0$ (i.e., $(i - \pi^e)\uparrow \Rightarrow I\downarrow$)

3 Ways to Finance

Debt - costs interest on principle
Equity - costs dividends paid to stockholders
Internal - use retained earnings; most popular form for U.S. firms

Why Invest? - trying to get some return on the money; have to measure against what that money could do otherwise; assuming stockholders would save the money (i.e., get a return on it), we compare present value of future profits associated with the investment to the amount needed for the investment

Addition to Profits - depends on...

Depreciation - profits decline over time

Diminishing Returns to Capital - profits decline as you add more of the same investment

Economic Conditions - expected future profits change based on expectations of economic condition (could go up or down)

PV of Profits = $\sum_{t=1}^{n} \frac{P_t}{(1+r)^t}$ (PV = present value)

Internal Rate of Return (IRR) -equivalent to treating investment like a saving's account; IRR is the interest rate that account would have (on average); found by solving PV of profits equation for r; benefit of IRR is you don't have to recompute PV when interest rate changes to make investment decision

Investment Decision -

PV - if PV of profit > cost of investment, do it
IRR - if IRR > cost of capital (interest rate), do it

Diminishing Returns

Depreciation

Economic Conditions (economy picks up; trucks not used much)

Example

		----------	Addition to profits		----------	PV of	
Project	**Cost**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Profit (7%)**	**IRR**
Truck 1	-45000	18000	18000	15000	12000	53944	16.2%
Truck 2	-45000	15000	15000	15000	12000	48519	10.5%
Truck 3	-45000	3000	6000	16500	30000	44400	6.6%
Truck 4	-45000	1500	3000	16500	24000	35801	0.0%
Desk 1	-7500	2700	2700	2700	2700	9145	16.4%
Desk 2	-7500	2250	2250	2250	2250	7621	7.7%
Computer	-4000	2100	1500	750	0	3885	5.1%
Loading Dock	-80000	16000	20000	24000	28000	73374	3.6%
Light Bulb	-1.5	1	1	0	0	1.81	21.5%

Internal Financing

Accelerator Model of Investment - not the same as doing multipliers which look at comparative statics (equilibrium conditions); this model instead looks at how investment changes over time so we see the in between steps, not just the end result; we'll look at two versions: one period and multiple periods (partial adjustment)

Internal Financing - firm is taking money that it would otherwise give to shareholders in order to increase future income for shareholders; this means investment is based on intertemporal budget constraint; investment lowers Y_1 in the hopes of increasing Y_2, Y_3, ... enough to increase consumption

$$C_1 + \frac{C_2}{1+r} + \frac{C_3}{(1+r)^2} + \cdots = Y_1 + \frac{Y_2}{1+r} + \frac{Y_3}{(1+r)^2} + \cdots$$

Alternative - assume shareholders have a risk free asset with yield i where they could put the money in; this is what we'll compare the investment's IRR to

Production Function, $Y = F(K,N)$ - since value of investment (i.e., IRR) depends on future income (Y), we can use the production function to link I to Y through K (capital)

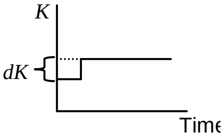
Limiting Factor - when we used $F(K,N)$ before we assumed K was fixed, now it's variable; to constrain the problem we'll assume the limiting factor is **expected demand**, Y^e

Capital (K) - capital stock this year = capital stock from last year plus addition to capital (i.e., investment) minus depreciation of last year's capital stock; δ = depreciation rate

$\mathbf{K_t = K_{t-1} + I_t - \delta K_{t-1}}$

Account for Inflation - if current price is P_t and there is a constant inflation rate π^e, then

$\mathbf{E(P_{t+i}) = P_t(1 + \pi^e)^i}$



Increase Capital Stock - to permanently increase capital stock by dK, we have to...

Pay for It Now - $-P_t\,dK$ (this is already a PV because it happens now)

Pay to Maintain - $-P_t(1 + \pi^e)(\delta dK)/(1 + i)^t$ (to get PV of Mx, need to add for t = 1,2,...)

Get More Income - change in capital leads to greater output Y based on $F_K = \partial F(K,N)/\partial K$; $P_t(1 + \pi^e)(F_K\,dK)/(1 + i)^t$ (want to add up PV for t = 1,2,...)

Put it together: ΔPV = $-P_t dK + \dfrac{P_t(1+\pi^e)(F_K dK - \delta dK)}{(1+i)} + \dfrac{P_t(1+\pi^e)^2(F_K dK - \delta dK)}{(1+i)^2} + \cdots$

Factor out $F_K\,dK - \delta dK$: ΔPV = $-P_t dK + (F_K dK - \delta dK)\left[\dfrac{P_t(1+\pi^e)}{(1+i)} + \dfrac{P_t(1+\pi^e)^2}{(1+i)^2} + \cdots\right]$

Real Interest Rate - $\mathbf{r \equiv (i + \pi^e)/(1 + \pi^e)}$; we usually estimate this by $i + \pi^e$

Geometric series simplifies (don't worry about it): $\mathbf{\Delta PV = -P_t\,dK + P_t(F_K - \delta)dK/r}$

Optimal Capital Stock (K^*) - set ΔPV = 0 and solve for K; two notes: (1) ΔPV levels off from diminishing returns; (2) K will be embedded in F_K; to solve for K, we need a specific production function:

$-P_t\,dK + P_t(F_K - \delta)dK/r = 0 \;\Rightarrow\; \mathbf{F_K = \delta + r}$ (or $F_K + \delta = r$)

Gross IRR - F_K; invest up to point that gross IRR equals **gross cost of capital**: direct cost of investment (δ) plus opportunity cost of investment (r)

Net IRR - $F_K + \delta$; invest up to point that net IRR equals **net cost of capital** (r)

Relate to Investment - use formula for capital above to solve for investment:

$$K_t = K_{t-1} + I_t - \delta K_{t-1} \;\Rightarrow\; I_t = \underbrace{K_t - K_{t-1}}_{\substack{\text{additional investment}\\ \text{to get to } K_t \text{ from } K_{t-1}}} + \underbrace{\delta K_{t-1}}_{\substack{\text{additional investment}\\ \text{to maintain } K_{t-1}}} = dK_t + \delta K_{t-1}$$

To get further results, we need a specific production function to find F_K

Cobb-Douglas Production Function - $Y = AK^{\alpha}N^{1-\alpha} \Rightarrow F_K = \alpha AK^{\alpha-1}N^{1-\alpha} = \alpha Y/K$

Optimal Capital Stock - $F_K = \alpha Y/K^* = \delta + r \Rightarrow K^* = \alpha Y/(\delta + r)$

Note: from here we can see directly that $Y\uparrow \Rightarrow I\uparrow$ and $r\uparrow \Rightarrow I\downarrow$

One Period Adjustment - assumes you invest enough to get from current capital stock (K^*_{t-1}, which we assume was optimal before) to new optimal value in a single period

$I_t = K^*_t - K^*_{t-1} + \delta K^*_{t-1} = K^*_t - (1 - \delta)K^*_{t-1} = \alpha Y_t^e/(\delta + r_t) - (1 - \delta)\alpha Y_{t-1}^e/(\delta + r_{t-1})$

Through a little algebra magic (adding and subtracting $\alpha Y_t^e/(r_{t-1} + \delta)$) we can "simplify" to:

$$I_t = \alpha\underbrace{\frac{Y_t^e - Y_{t-1}^e}{\delta + r_{t-1}}}_{\Delta Y} - \alpha\underbrace{\frac{(r_t - r_{t-1})Y_t^e}{(\delta + r_t)(\delta + r_{t-1})}}_{\Delta r} + \alpha\frac{\delta Y_{t-1}^e}{(\delta + r_{t-1})} \longrightarrow$$

Replacement Investment - I when $r_t = r_{t-1}$ and $Y_t^e = Y_{t-1}^e$; additional investment to maintain capital stock so $K_t = K_{t-1}$

Example - $\alpha = 0.25$, $\delta = 0.05$, $r = 0.05 \Rightarrow I_t = \underbrace{2.5\Delta Y^e}_{\Delta Y = \$1B \Rightarrow I\uparrow \text{ by } 2.5GDP} - \underbrace{25\Delta r Y_t^e}_{\Delta r = -1\% \Rightarrow I\uparrow \text{ by } 2.5GDP} + 0.125Y_{t-1}^e \longrightarrow$

I should be about 12.5% in the long-run; but short-run effects can be huge

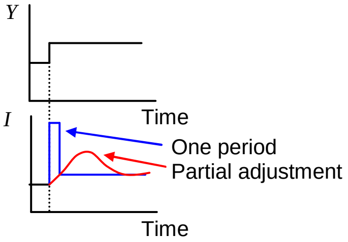
Example - oven makes 1 loaf/hour; limited to 50 hr/week and lasts 1000 hr; Y^e = 1000/wk; K^* = 1000/50 = 20 ovens; in long-run, expect to replace 1 oven per week (I = 1); if $Y^e\uparrow$ to 1100 (10% increase), now K^* = 22 ovens so investment includes 1 oven for depreciation plus 2 new ovens (I = 3)... that's 200% increase; long-run will be 1.1 ovens/week

Realistic? - average investment about 12.5% with large variations from average... that's realistic, although amount of variation with on period adjustment may be too much

Partial Adjustment - assumes you only invest a fraction (λ) of the gap between current capital stock and new capital stock; eqns included for completeness (don't need to know them)

$I_t = \lambda(K^*_t - K^*_{t-1}) + \delta K^*_{t-1}$, where $K_t = \lambda K^*_{t-1} + \lambda(1 - \lambda)K^*_{t-2} + \lambda(1 - \lambda)^2K^*_{t-3} + \ldots$

$I_t = \delta K_t + \lambda(K^*_t - K^*_{t-1}) + \lambda(1 - \lambda)(K^*_{t-1} - K^*_{t-2}) + \lambda(1 - \lambda)^2(K^*_{t-2} - K^*_{t-3}) + \ldots$



Link to Recession? - $Y^e\downarrow$ (recession) and $r\downarrow$ (expansionary monetary policy) have opposite effect so I may $\uparrow$ or $\downarrow$; I won't $\uparrow$ with certainty until $Y^e\uparrow$ (with r unchanged)

Cause? - so is recession caused by $I\downarrow$ or does $I\downarrow$ because of recession ($Y^e\downarrow$); it's a self-fulfilling prophesy: if firms anticipate $Y^e\downarrow$, then $I\downarrow$ which will causes $Y\downarrow$ even if it wasn't going to happen!

Other Methods of Financing

Example - $10,000 investment now pays $1,500 per year for 10 years.

Internal Financing - decision depends on real interest rate at which stockholders can reinvest their money; r_{TB} (for Treasury Bills); if r_{TB} = 10%

$$\Delta\text{PV} = -10000 + \frac{1500}{1.1} + \frac{1500}{(1.1)^2} + \cdots + \frac{1500}{(1.1)^{10}} = -783 \;\therefore\; \text{don't do project}$$

Debt Financing - decision depends on interest rate firm gets to borrow at (r_{DEBT}); it will be higher than r_{TB} to compensate lenders for the risk that the firm doesn't pay back the loan; for this example, let's assume there's no extra risk so $r_{\text{TB}} = r_{\text{DEBT}}$ = 10%; in this case firm doesn't pay anything in year 1; it pays interest on the loan each year (10% of 10,000 =

1,000); then in year 10, the firm my pay back the amount borrowed ($10,000); the change in the present value of income for shareholders in the same

$$\Delta\text{PV} = 0 + \frac{1500-1000}{1.1} + \frac{1500-1000}{(1.1)^2} + \cdots + \frac{(1500-1000)-10000}{(1.1)^{10}} = -783$$

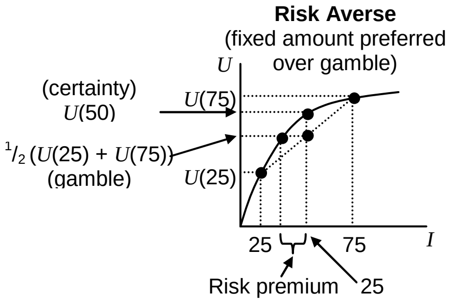
Equity Financing - new stockholders take share of firm's profits (not just from investment being financed and not just for duration of project); for now assume $r_{\text{EQUITY}} = r_{\text{TB}} = 10\%$; in year 1 the firm pays nothing, but firm pays 1000 every period; investment decision is the same as other forms of financing

$$\Delta\text{PV} = 0 + \frac{1500-1000}{1.1} + \frac{1500-1000}{(1.1)^2} + \cdots + \frac{(1500-1000)}{(1.1)^{10}} - \frac{1000}{(1.1)^{11}} - \frac{1000}{(1.1)^{12}} - \cdots = -783$$

Summary - investment decision is identical regardless of method of financing if the rates are the same; in reality, however, $r_{\text{TB}} < r_{\text{DEBT}} < r_{\text{EQUITY}}$, but investment decision is still the same because the differences in interest rates are accounting for differences in risk

Risk & Return

Basic Gamble - say you have $50,000; if you wager $25,000 on an even with 0.50 probability which results your losing or gaining $25,000 (i.e., total available to you is $25,000 or $75,000)



Expected Utility - probability of each outcome times the utility of each payoff in that outcome (e.g., $E(U) = 0.5U(25\text{K}) + 0.5U(75\text{K})$)

Risk Averse - given the choice between a fixed amount of money and a gamble with an expected payoff of the same amount, a risk averse person would take the fixed amount; this is because of "diminishing returns" (i.e., concave utility function)

Example - $U(C) = C^{1/2}$

Fixed amount (no gamble) - U(50K) = 50K1/2 = 233

Gamble - E(U of gamble) = 1/2U(25K) + 1/2U(75K) = 1/2(158) + 1/2(274) = 216

∴ person with this utility function would take the fixed amount (i.e., risk averse)

Risk Premium - can look at it as either the amount by which the fixed amount needs to be lowered, or the amount by which the expected payoff of the gamble needs to be raised in order to get a risk averse person to take the gamble; actual value will be depend on the utility function (i.e., level of risk aversion); can write as $ or %

Example - want $pU(25\text{K}) + (1-p)U(75\text{K}) = U(50\text{K}) = 233$; solve for p:

$p(U(25\text{K}) - U(75\text{K})) = U(50\text{K}) - U(75\text{K}) \Rightarrow p = (U(50\text{K}) - U(75\text{K}))/(U(25\text{K}) - U(75\text{K})) =$ 0.434 (lots of rounding error if you just use 233, 274 and 158 for the utilities)

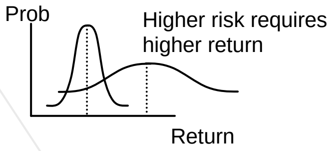
$ Value - E($ of gamble) = 0.434(25K) + 0.566(75K) = **$53,300**

% Value - (53,300 - 50000)/50000 = **6.6%**

More Risk Averse - more concave the utility function, the more risk average, hence the higher risk premium (e.g., if $U(C) = C^{1/5}$, risk premium is 10.5%)

Risk Neutral - $U(C) = C$ (linear); risk premium is 0%

Graphically - need higher payoff to make up for increased risk:



Modigliani Miller Theorem

MM Theorem - if firms have same expected stream of income and same variance (risk), then market value of equity plus debt is constant (e.g., firm 1 has no debt and firm 2 has debt D_2: $V_1 = V_2 + D_2$)

- **Example** -
  - **Firm A**: 0.5 chance of \$8K/year and 0.5 chance of \$12K/year; expected earnings 10K/year (± 20%); expected earnings over lifetime (r_{TB} = 10%) is 10K/0.1 = \$100K; risk averse investor would be willing pay less than this; assume risk premium is 2% so value of firm is V_A = 10K/12% = <u>\$83,333</u>
  - **Firm B**: required rate of return 12% (same as Firm A); expected earnings of \$10K ± 20% (same as Firm A), but has \$50K debt (at 10%); ∴ income is actually 0.5 chance of \$8K - \$5K = \$3K and 0.5 chance of \$12K - \$5K = \$7K; expected income is \$5K ± 40% (i.e., debt adds risk), use MM Theorem to calculate value of firm: $V_B + 50K = V_A \Rightarrow V_B$ = 83,333 - 50K = <u>\$33,333</u>; required return to account for risk: 5K/r = 33,333 ⇒ r = <u>15%</u>
  - **Firm C**: required return is 12% and expected earnings are \$4K/year; given V_C = 4K/0.12 = \$33,333, what is risk class? based on info above a 12% return is required for ± 20%, ∴ payoffs for this firm are 0.5 chance of 3200 and 0.5 chance of 4800 because 800 is 20% of \$4K
  - **Firm D**: Add \$50K capital to firm C to raise earnings to \$10K/year without changing risk (i.e., still ± 20%)
    - **Debt Financing** - becomes same scenario as firm B so V_{D1} = \$33,333; since there is no change in value of firm (vs. firm C), stockholders are indifferent to the investment
    - **Equity Financing** - V_{D2} = \$83,333 = \$33,333 + \$50K (equity); old shares still worth \$33,333 so original stockholders are not better off
    - **Paradox?** - 10% for debt financing and 12% for equity financing came to same investment decision; this is because difference in risk between different financing options; for debt financing, bank doesn't assume any of the risk (always gets its 10%); for equity financing, new stockholders have to bear some of the risk (don't collect if firm doesn’t make enough profit)

Money Demand

Money Demand - $L(Y,i)$; $L_Y > 0$ & $L_i < 0$; at equilibrium = M/P

What is Money? - has multiple meanings:

- "How much money do you make?"... income (flow)
- "How much money are you worth?"... wealth (stock)
- **Economic Definition** - liquid portion of wealth (cash, checking balances, etc.)
  - **Liquid** - can be used for transactions

Example - person A has most income, B has most wealth, C has most money

Sum to wealth

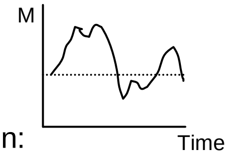
	A	B	C
M	1,000	2,000	3,000
Home	90,000	--	50,000
Securities	5,000	220,000	--
Wealth	96,000	222,000	53,000
Income	60,000	40,000	20,000

M1 - purely transaction-based definition; currency plus checking account balances (demand deposits) & travelers checks; ~ $650B in cash & $650B in others; total ~ $1.3 Trillion)

M2 - purely transaction-based (M1) plus easily transferable savings accounts (e.g., overnight repurchase agreements, US dollar accounts in Europe, money-market mutual funds, savings deposits, small time deposits); ~ $6 Trillion

M3 - everything in M2 plus large time deposits & other accounts used less frequently for transactions purposes; ~ $9 Trillion

Credit Cards - affect how much money people want to hold, but are excluded from definition of money because they're not assets



Real Income - Y = nominal income ÷ price level (P)

Nominal Income - PY

Velocity of Money - number of times a dollar gets spent in a year; $V = PY/M$; depends on:

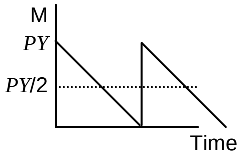
1. Frequency of paychecks (# paychecks↑ ⇒ M↓ & V↑)
2. Regularity of paychecks (more regular (i.e., less seasonal) ⇒ M↓ & V↑)
3. Predictability of paychecks (more predictable ⇒ M↓ & V↑)
4. Ease of credit (more credit ⇒ M↓ & V↑)

Historic Trends - V for M2 < V for M1 (because M2 has more money); V for M2 hasn't changed much over time (people still save roughly same % of income; V for M1 has been increasing (more credit, less seasonal, & more frequent pay periods)

	1900	1930	1945	1960	1980	2000
M1	--	2.4	2.0	3.0	6.9	8.8
M2	2.5	1.5	1.4	1.7	1.8	1.8

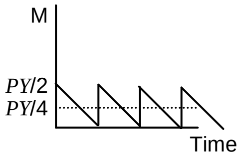
(can't distinguish)

Intuition Behind Transaction Demand - look at amount of money people hold over time; fluctuates based on Y and i; transaction models try to explain how based on how the money is used

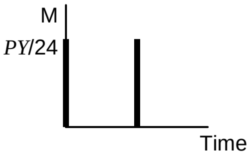


Single Pay Period - assume person gets paid PY at start of year and spends it all continuously over the course of the year; on average, person holds $PY/2$; expand concept to entire economy (i.e., PY = nominal GDP) and the amount of money needed is M = $PY/2$ $\therefore V = PY/PY/2 = 2$

- **Two Pay Periods** - assume person gets paid twice a year ($PY/2$ each time) at start of year and again in 6 months; spends it all continuously over the six months; on average person holds $PY/4$; expand concept to entire economy and the amount of money needed is M = $PY/4$ ∴ $V = PY/PY/4 = 4$

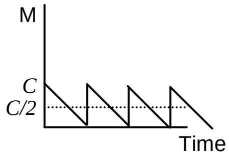


- **All Credit** - Assuming person gets paid twice a month; each pay period, uses all income to pay off credit card; all purchases during rest of month are on the credit card; amount of money needed is very low and velocity is very high



Baumol Model

Inventory Model - based on transaction demand for money; looks at keeping money like stocking inventory; short on intuition, but empirically testable
- **Receipts (*T*)** - assume receipts (income) = expenditures; T in real dollars ∴ PT is nominal
- **Withdrawal (*C*)** - amount of usual withdrawal (from M2 to cash); depends b & i; determines amount of money needed M = $C/2$
- **Cost of Withdrawal (*b*)** - real cost of making a withdrawal
- **Interest (*i*)** - cost of holding cash (instead of letting it earn interest)



Number of Withdrawals - PT/C

Average Cash on Hand - $C/2$

Total Nominal Cost - $\frac{PT}{C}Pb + i\frac{C}{2}$; cost of making withdrawals plus cost of interest foregone
- **Trade-off** - to lower cost of withdrawals you should make fewer of them (i.e., C↑), but that increases the interest foregone

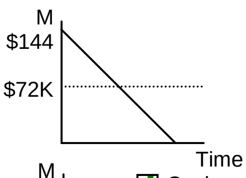
Minimize Cost - decision variable is C so take first derivative wrt C and set it equal to zero:

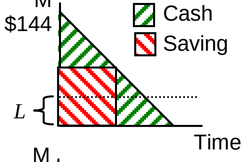
$$-\frac{PT}{C^2}Pb + \frac{i}{2} = 0 \;\Rightarrow\; C^* = P\sqrt{\frac{2bT}{i}}$$

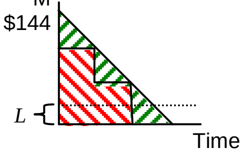
Money Supply - M = $\frac{C^*}{2} = \sqrt{\frac{bT}{2i}}$; we can substitute GDP (Y) for T to get money supply
- **Results** - money supply changes in proportion to price level (P↑ by 10% ⇒ M↑ by 10%); M increases by less than in proportion to income and decreases by less than in proportion (specifically by $Y^{1/2}$ and $i^{-1/2}$)
- **Problem** - can't really substitute GDP because some transactions require money but aren't captured in GDP (used cars, illegal activities, etc.)

Tobin Model

Scenario - assume T = $144K/year; can put money in interest bearing asset that earns i = 10%; each transaction costs b = $700
- **No Bank Trips** - demand for money L = $144K/2 = $72K; no interest earned and no costs incurred
- **1 Trip** - save 1/2 money at start of year and go back to back to get it in 6 months; L = (T/2)/2 = $36K; 2 transactions so cost is 2*$700 = $1400; earn interest on half the money for half the year: $T/2 \cdot i \cdot 1/2$ = $3600
- **2 Trips** - save 2/3 money at start of year; go back to get 1/2 of savings (T/3) in 4 months (1/3 of year); go back again to get rest at 2/3 year; L = (T/3)/2 = $24K; transactions cost 3*$700 = $2100; 1/3 of money earns interest for 2/3 year and 1/3 of money earns interest for 1/3 of year: $T/3 \cdot i \cdot$ (1/3+2/3) = $4800







***n* Trips** -

$L = (T/n)/2$

Cost = bn

Interest Earned = $(T/n)\cdot i\cdot n\cdot(n-1)/2$ (Note: sum of t consecutive integers is $t(t+1)/2$)

Decreasing Returns - each additional trip yields less additional interest income

Goal - maximize net benefit (interest earned minus transactions cost); in this scenario, best to make 3 trips so L = $18K

Trips	TX	Cost	Interest	Net	L
0	0	0	0	0	72000
1	2	1400	3600	2200	36000
2	3	2100	4800	2700	24000
3	**4**	**2800**	**5400**	**2600**	**18000**
4	5	3500	5760	2260	14400

Results -

$i\uparrow \Rightarrow$ earnings increase so number of transactions increase... $L\downarrow$

$T\uparrow \Rightarrow L\uparrow$ (regardless of # of transactions); earnings increase so number of transactions increase; <u>manage cash more intensively</u>; hold more money but less than in proportion to ΔT (similar to Baumol model)

Economies of Scale - same cost per transaction so it pays more (on net) to make more transactions when you have more money

Problem - why don't we do this? Get paid twice per month so T very low; doesn't pay to cover transaction cost

Second Problem - getting paid twice per month implies we would hold a week's income in cash which implies $V = 52$, but it's only 8.8 (we hold 5 weeks income in cash); why?

1. Households vs. firms (firms hold more cash)
2. Most cash is $100 bills... probably held for illegal transactions or overseas
3. Portfolio

    Quantity Theory of Money - by Milton Friedman; large amounts of cash are held for portfolio reasons, not just transactions

Empirical Work

Allan Meltzer, "The Demand for Money: Evidence From the Time Series." JPE 1963

Data - annual data from 1900-1950

Logs - use ln(M/P), ln(i), ln(Y), etc. because coefficients can be interpreted as elasticities

Baumol Model - suggests ln(M/P) = [const] - 0.5ln(i) + 0.5ln(Y)

Results -

$\ln(\text{M1}/P) = [\text{const}] - \underset{(0.04)}{0.99}\ln(i) + \underset{(0.03)}{1.11}\ln(W/P)$ $R^2 = 0.992$ (W = wealth)

$\ln(\text{M2}/P) = [\text{const}] - \underset{(0.05)}{0.5}\ln(i) + \underset{(0.02)}{1.32}\ln(W/P)$ $R^2 = 0.994$

$\ln(\text{M1}/P) = [\text{const}] - \underset{(0.04)}{0.79}\ln(i) + \underset{(0.04)}{1.05}\ln(Y)$ $R^2 = 0.981$

$\ln(\text{M1}/P) = [\text{const}] - \underset{(0.05}{0.92}\ln(i) + \underset{(0.10)}{0.97}\ln(W/P) + \underset{(0.09)}{0.13}\ln(Y)$ $R^2 = 0.995$ Y not significant

Notes -

1. Makes sense to use M2 with wealth (portfolio) and M1 with income (transactions)
2. W more important than Y for determining money demand

3. Multicollinearity problem; Y & M correlated, but so are W & Y; can't interpret coefficients
4. M/P always gets good fits, but not always good forecasts
5. Coefficient for i is always < 0 and significant; coefficient for scale (W or Y) is always > 0 & significant

Baby Sitting Example

Sweeney & Sweeney, "Monetary Theory and the Great Capitol Hill Baby Sitting Co-op Crisis."

Background - cooperative of 150 families taking advantage of economies of scale in baby-sitting; true cost s sitting there watching "Clifford" so adding another kid doesn't cost much on the margin

Barter - economists usually think of barter as being inefficient because it requires **double coincidence of wants** (have to find someone who wants what you're supplying and has what you want; usually takes along time to find someone to trade with)

Potential Problems - aside from inefficiency of barter, people could abuse the system (use baby sitter 20 times and only sit 5 times)

Solutions -

(1) **bookkeeping** - credit and debit hours of babysitting; problems with accuracy and unpleasant phone calls

(2) **scrip** - explain rules and give 40 scrip ("money") good for 1/2 hour of babysitting; people use/earn scrip with no bookkeeping required; problems include counterfeiting and people moving (decrease money supply)

$L > M/P \Rightarrow$ people baby sit to earn more scrip

$L < M/P \Rightarrow$ people go out more to get rid of scrip

Crisis - CHBSC used scrip; co-op had administration (people to explain rules, interview new members, etc.); wouldn't work with volunteers so staff was paid with scrip through dues (4200 scrip/year ~ 2100 hours ~ 1hr/mo/couple... call that T = net taxes); problem was staff being too efficient; only used 3800 scrip ~ 1900 hours... call that G = government purchases; money supply was shrinking; result - more people on the list to baby-sit, but fewer people asking for baby sitters (S > D; equivalent of unemployment)

Economist Solution - have administrators redistribute the "surplus" (monetary policy); not obvious to people that money supply is the problem

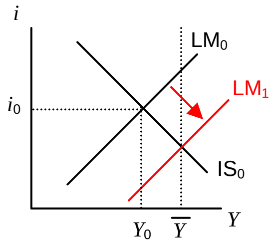
CHBSC Solution - rule requiring people to go out at least once every six months; most of the members were lawyers and used to "Stalinist, central planning" (Bomberger)

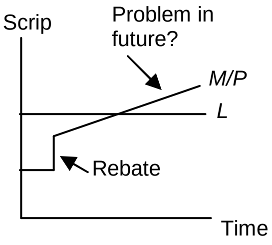
IS-LM - what's needed is either increasing the money supply ($M\uparrow$) or have prices drop ($P\downarrow$); problem is prices are fixed (scrip says "1/2 hour")

Black Market - could have baby sitters offering to work for less ("I'll work for 1 scrip per hour"); could have people going outside the co-op spending real money rather than scrip

Keynesian Idea - monetary is great compared to barter, but has problems when money market is out of balance; need a central bank to maintain $L = M/P$ or need flexible pricing so P can change to maintain balance (problem with sticky prices... especially wages)

Eventual Solution - CHBSC redistributed the "surplus" and gave new members 60 units of scrip (rather than 40); also, members leaving the co-op only have to return 40 units; now have increasing money supply... could cause problems in future





History of Economic Thought on Money

John Maynard Keynes - not first to use money demand or first to talk about recessions; first to use monetary policy to explain recession

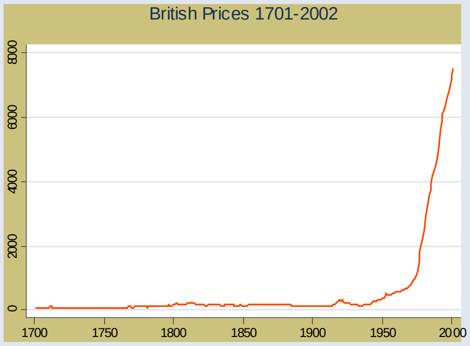
"Common Knowledge" - "downturns are a necessary evil in capitalist economy"; capitalist system is best because it's increases standard of living fastest, but downturns are "necessary" because doesn't seem to be anything to do about it; Keynes was unique because he was the only one not saying recessions would get worse

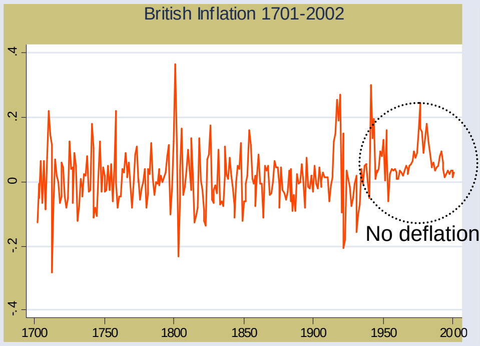
Minority Opinions -

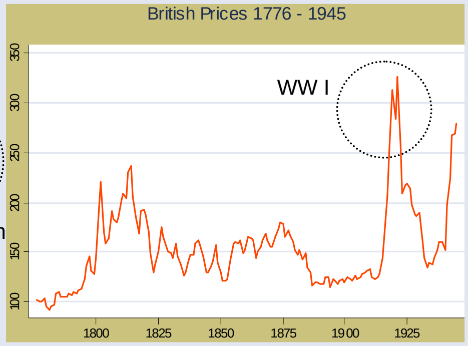
- **Marx** - downturns are good; capitalist system produces too much then cuts back to get rid of inventories; owners take advantage of workers; said downturns would get worse and worse, but increased dependence of technology would result in educated work force that would eventually overthrow the owners; "Centralization of the means of production and socialization of labor at last reach a point where they become incompatible with their capitalist integument." "Thus integument is burst asunder. The knell of capitalist private property sounds. The expropriators are expropriated."
- **S?????** - government help doesn't let firms eliminate "dead wood" so downturns would get worse over time; “[A recovery] is sound only if it [comes] of itself. For any revival which is merely due to the artificial stimulus leaves part of the work of depressions undone and adds, to an undigested maladjustment, new maladjustment of its own which has to be liquidated in turn, thus threatening business with another [worse] crisis ahead."

Britain - looking at money since 1700s shows not much change until last 50 years, and since then there's been no deflation

- **1776-1900** - price fairly stable because on gold standard; (1) government can't just print money (doesn't have enough gold to back it up); (2) fixes exchange rate between pound and any other currency on gold standard (1 pound = 5 U.S. dollars); designed to prevent persistent inflation


British Prices 1701-2002


British Inflation 1701-2002


British Prices 1776 - 1945

- **WW I** - went off gold standard; tripled money supply which tripled prices
- ***Economic Consequences of the Peace*** - published by Keynes in 1919; said reparations on Germans were shortsighted; just making German democracy unpopular (people would view it as collecting money for the Allies); led to hyperinflation like Keynes predicted
- **After War** - went back to gold standard; had two options: (1) change exchange ratio (i.e., don't reverse the inflation), (2) cut money supply to get back to old exchange ratio; arguments for latter case were to gain credibility for future and to keep financial markets from moving to New York

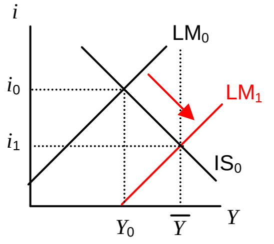
Economic Consequences of Mr. Churchill - published by Keynes in 1925; tried to gain credibility by using similar title to previous paper where he was correct about Germany; argued that going to gold standard at current level would be better

Depression - Britain chose latter policy and had rapid deflation; unemployment was very high; Britain had long depression before the world-wide Great Depression hit in the 30s; people started to argue that Marx was right; Keynes argued it was a monetary problem in his book

A General Theory of Employment, Interest and Money - published by Keynes in 1936; didn't write it as an "I told you so" book because he still wasn't entirely convinced; prices adjusted quickly after Napoleonic (1812) War; Keynes thought deflation would take much longer after WW I because of unions (sticky prices); it happened faster than Keynes thought, but still didn't have unemployment problem solved

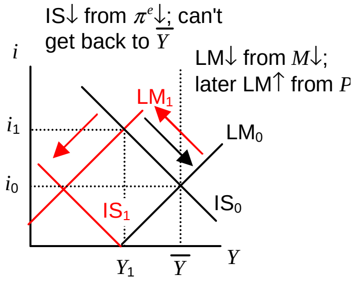
General Idea of Monetary Policy - $P\downarrow \Rightarrow M/P\uparrow \Rightarrow \text{LM}\uparrow \Rightarrow i\downarrow \Rightarrow I\uparrow \Rightarrow Y\uparrow$; key is lower interest rates increasing output via investment

Liquidity Trap - if potential GDP is out too far, monetary policy alone can't solve the problem; eventually get to $i = 0$ from decreasing price level (or increased money supply), but will never get to full employment; once interest rates go to zero, investment is no longer encouraged and money gets "trapped" in portfolios (people hold money rather than invest it)



Pigou Effect - Pigou argued for $C(Y - T, i, \text{Wealth})$; deflation increases wealth which increases consumption; problem is effect of wealth on consumption (and hence output) is weak compared to effect of interest rate on investment (and hence output)

Getting Trapped - $I(i - \pi^e)$; suppose i = 4% and π^e = 2% $\therefore$ real rate = 2%; cut money supply to get to previous exchange rate (move to LM$_1$); wait for deflation to get LM back to full employment, but if it takes too long, people expect deflation so π^e drops (i.e. move to IS$_1$; people expect to pay back loans with money that is worth more so they don't want to borrow); by the time prices fall to original LM curve (i = 4%), still haven't reach potential GDP because real rates are too high



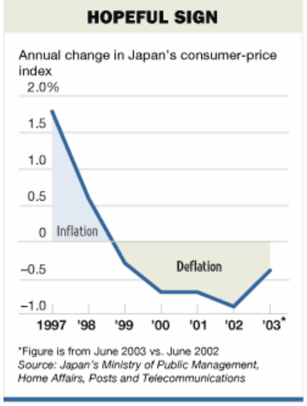
Solution - Keynes argued in this case monetary policy is no good so we need fiscal policy (G↑) to bring economy back; when in a liquidity trap, fiscal policy is fine to use because there is not much crowding out

Keynesian Policy - used as pejorative term for <u>any</u> fiscal policy (usually deficit spending) used to stimulate the economy; **Note:** Keyes argued for monetary policy; only talked about fiscal policy when interest rates were too low for monetary policy to work

Why Study This? - Japan; 1990s look the same; interest rates near zero and high unemployment

US? - target federal funds rate at 1%; lowest in 40 years; been there for a while; if it stays too long, we could have $\pi^e\downarrow$ and end up in liquidity trap

Bomberger - looks like some inflation is good for a monetary policy cushion... maybe 4%



HOPEFUL SIGN

Annual change in Japan's consumer-price index

*Figure is from June 2003 vs. June 2002

Source: Japan's Ministry of Public Management, Home Affairs, Posts and Telecommunications

RISKY CYCLE

Prices are starting to fall...

Year-to-year percentage change in consumer prices

LOWER RATES

Sources: Federal Reserve; WSJ Statistics

Missing Money

Goldfield, "The Case of the Missing Money."

- **Demand for Money (L)** - can't really predict it, but we assume it's function of Y and i and that $L_Y > 0$ and $L_i < 0$; assume it's: $\ln(L)_t = a_0 + a_1\ln(Y)_t + a_2\ln(i)_t + \ldots$
- **Predict L** - usually use $L = M/P$ (implicitly assumes market is at equilibrium); Goldfield argued that money demand may adjust slowly because there are two components to the cost of reaching equilibrium:
  - **Partial Adjustment** - cost of changing L_t is $a(\ln(M/P)_t - \ln(L)_t)^2 + b(\ln(M/P)_t - \ln(M/P)_{t-1})^2$

    Cost of holding non-optimal amount of money | Adjustment cost; cost to change portfolio
    - **Holding Cost** - squared because (1) holding too much is just as bad as holding too little, (2) cost grows the farther you are from optimal amount
    - **Adjustment Cost** - squared because (1) adjusting cost in both directions, (2) cost of adjustment grows the faster you change
    - **Min Cost** - take derivative wrt $\ln(M/P)_t$, set it equal to zero and solve for $\ln(M/P)_t$

      d(cost)/d($\ln(M/P)_t$) = 2a($\ln(M/P)_t - \ln(L)_t$) + $2b(\ln(M/P)_t - \ln(M/P)_{t-1}) = 0$

      $$\ln(M/P)_t = \frac{a}{a+b}\ln(L)_t + \frac{b}{a+b}\ln(M/P)_{t-1} = \lambda\ln(L)_t + (1-\lambda)\ln(M/P)_{t-1}$$
      - **Zero Adjustment Cost** - note if $b = 0$, $\ln(M/P)_t - \ln(L)_t$ (i.e., adjust immediately)
    - **Model** - substitute in formula for $\ln(L)_t$ and get:

      $\ln(M/P)_t = \lambda a_0 + \lambda a_1\ln(Y)_t + \lambda a_2\ln(i)_t + \ldots + (1-\lambda)\ln(M/P)_{t-1}$

      Rewrite with single parameters (that's what we'll estimate in a regression):

      $\ln(M/P)_t = b_0 + b_1\ln(Y)_t + b_2\ln(i)_t + \ldots + c\ln(M/P)_{t-1}$
      - **Interpreting Coefficients** - note that $1 - \lambda = c$ (or $\lambda = 1 - c$); now $b_1 = \lambda a_1 \Rightarrow a_1 = b_1/(1-c)$; similarly $a_2 = b_2/(1-c)$
        - **"Short-Run" Elasticities** - b_1, b_2, etc. (1 quarter)
        - **"Long-Run" Elasticities** - a_1, a_2, etc. (eventual)
- **Goldfield's Results** - quarterly data 1952:2 to 1973:4

  $\ln(\mathrm{M1}/P)_t$ = [const] + $0.179\ln(Y)_t - 0.042\ln(i_{\mathrm{td}})_t - 0.181\ln(i_{\mathrm{cp}})_t + 0.676\ln(\mathrm{M1}/P)_{t-1}$

  $R^2 = 0.995$ (0.04) (0.011) (0.003) (0.068)

  $\ln(\mathrm{M2}/P)_t$ = [const] + $0.206\ln(Y)_t - 0.021\ln(i_{\mathrm{tb}})_t - 0.071\ln(i_{\mathrm{sl}})_t - 0.029\ln(i_{\mathrm{td}})_t + 0.884\ln(\mathrm{M2}/P)$

  $R^2 = 0.999$ (0.054) (0.003) (0.028) (0.012) (0.045)
- **Problem** - good fit doesn't imply good forecast; forecast error for 1976:2: -22% for M1 and -4.4% for M2
  - **Reason** - overestimated real money balances because he underestimated inflation; problem wasn't too little money, but too much inflation
    - **Example** - $\mu_i = (d\mathrm{M}i/dT)/\mathrm{M}i$ = rate of growth of Mi (i = 1 or 2); π_i = inflation rate predicted implicitly by Goldfield's model; π = actual inflation rate

	μ_1	μ_2	π_1	π_2	π
1973	4.9%	5.7%	5.2%	5.5%	9.1%
1974	4.8%	5.8%	5.5%	6.7%	11.4%
1975	5.2%	11.4%	3.7%	7.3%	7.1%

- **Lesson** - overestimating real money balances is equivalent to underestimating inflation; problem wasn't missing money, but too much inflation

The Federal Reserve

Required Reserve Ratio - amount of money banks are required to hold in cash at the end of each day as percentage of total deposits

Federal Funds Rate - interest rate banks charge each other to borrow money (usually lent daily to cover RRR requirement)

Open Market Operations - buy/sell government securities in order to achieve monetary policy objectives (manipulate M or FFR)

Expansionary - buy bonds ⇒ M↑ ⇒ banks have more cash (excess reserves) ⇒ FFR↓

Contractionary - sell bonds ⇒ M↓ ⇒ banks have less cash (harder to make RRR) ⇒ FFR↑

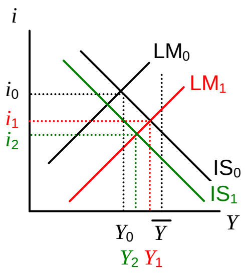
Money Supply Target - Fed focused on (dM/dt)/M (growth rate of money supply) and set targets for M1, M2, and M3 growth; usually only concerned about a range for FFR (used in 80s)

Interest Rate Target - Fed sets very narrow range for FFR and doesn't worry about money supply (current policy)

Potential Problems - unforeseen problems cause problems with monetary policy; say Fed targets specific ΔM or wants FFR = i_1 to get to Y_1 (and i_1)

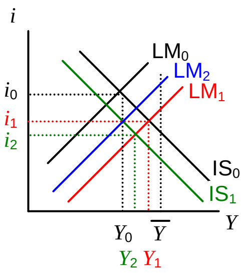
(1) IS↓ (for any reason, but usually π^e↓ or I↓)

Money Supply Target



IS↓ keeps Fed from target; i & Y lower

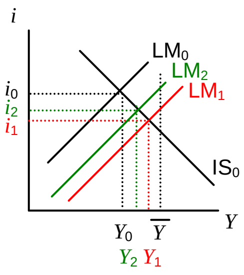
Interest Rate Target



IS↓ makes Fed lower ΔM to keep on I target; actual Y even lower than with money supply target

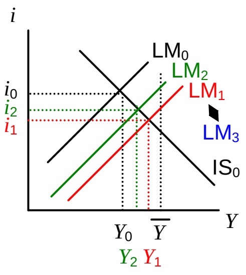
(2) LM↓ (from L↑)

Money Supply Target



L↑ (LM↓) cancels part of Fed's ΔM; i higher & Y lower than target

Interest Rate Target



L↑ (LM↓) has no effect because Fed focused on i; gets to target (takes care of "missing money" problem)

Summary - if IS is more unpredictable, use money supply target because interest rate target exaggerates ΔY from ΔIS (that's bad... recessions worse); if L (LM) is more unpredictable, use interest rate target because money supply target is countered by ΔY from ΔLM (policy is ineffective... not as bad as the exaggeration problem with IS)

Real World - usually (at least currently) L is more difficult to predict so Fed focuses on target FFR

Money Demand

Money Demand - $L(Y,i)$; $L_Y > 0$ & $L_i < 0$; at equilibrium = M/P

What is Money? - has multiple meanings:

- "How much money do you make?"... income (flow)
- "How much money are you worth?"... wealth (stock)
- **Economic Definition** - liquid portion of wealth (cash, checking balances, etc.)
  - **Liquid** - can be used for transactions

Example - person A has most income, B has most wealth, C has most money

Sum to wealth

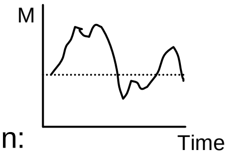
	A	B	C
M	1,000	2,000	3,000
Home	90,000	--	50,000
Securities	5,000	220,000	--
Wealth	96,000	222,000	53,000
Income	60,000	40,000	20,000

M1 - purely transaction-based definition; currency plus checking account balances (demand deposits) & travelers checks; ~ \$650B in cash & \$650B in others; total ~ \$1.3 Trillion)

M2 - purely transaction-based (M1) plus easily transferable savings accounts (e.g., overnight repurchase agreements, US dollar accounts in Europe, money-market mutual funds, savings deposits, small time deposits); ~ \$6 Trillion

M3 - everything in M2 plus large time deposits & other accounts used less frequently for transactions purposes; ~ \$9 Trillion

Credit Cards - affect how much money people want to hold, but are excluded from definition of money because they're not assets



Real Income - Y = nominal income ÷ price level (P)

Nominal Income - PY

Velocity of Money - number of times a dollar gets spent in a year; **$V = PY/M$**; depends on:

1. Frequency of paychecks (# paychecks↑ ⇒ M↓ & V↑)
2. Regularity of paychecks (more regular (i.e., less seasonal) ⇒ M↓ & V↑)
3. Predictability of paychecks (more predictable ⇒ M↓ & V↑)
4. Ease of credit (more credit ⇒ M↓ & V↑)

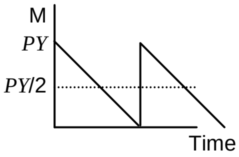
Historic Trends - V for M2 < V for M1 (because M2 has more money); V for M2 hasn't changed much over time (people still save roughly same % of income; V for M1 has been increasing (more credit, less seasonal, & more frequent pay periods)

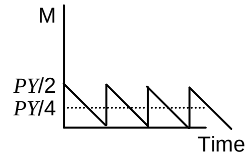
	1900	1930	1945	1960	1980	2000
M1	--	2.4	2.0	3.0	6.9	8.8
M2	2.5	1.5	1.4	1.7	1.8	1.8

(can't distinguish)

Intuition Behind Transaction Demand - look at amount of money people hold over time; fluctuates based on Y and i; transaction models try to explain how based on how the money is used

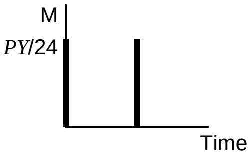
Single Pay Period - assume person gets paid PY at start of year and spends it all continuously over the course of the year; on average, person holds $PY/2$; expand concept to entire economy (i.e., PY = nominal GDP) and the amount of money needed is M = $PY/2$ $\therefore V = PY/PY/2 = 2$





Two Pay Periods - assume person gets paid twice a year (*PY*/2 each time) at start of year and again in 6 months; spends it all continuously over the six months; on average person holds *PY*/4; expand concept to entire economy and the amount of money needed is M = *PY*/4 ∴ *V* = *PY*/*PY*/4 = 4

All Credit - Assuming person gets paid twice a month; each pay period, uses all income to pay off credit card; all purchases during rest of month are on the credit card; amount of money needed is very low and velocity is very high



Baumol Model

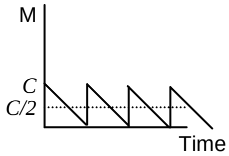
Inventory Model - based on transaction demand for money; looks at keeping money like stocking inventory; short on intuition, but empirically testable

Receipts (*T*) - assume receipts (income) = expenditures; *T* in real dollars ∴ *PT* is nominal

Withdrawal (*C*) - amount of usual withdrawal (from M2 to cash); depends *b* & *i*; determines amount of money needed M = *C*/2

Cost of Withdrawal (*b*) - real cost of making a withdrawal

Interest (*i*) - cost of holding cash (instead of letting it earn interest)



Number of Withdrawals - *PT*/*C*

Average Cash on Hand - *C*/2

Total Nominal Cost - $\frac{PT}{C}Pb + i\frac{C}{2}$; cost of making withdrawals plus cost of interest foregone

Trade-off - to lower cost of withdrawals you should make fewer of them (i.e., *C*↑), but that increases the interest foregone

Minimize Cost - decision variable is *C* so take first derivative wrt *C* and set it equal to zero:

$$-\frac{PT}{C^2}Pb + \frac{i}{2} = 0 \;\Rightarrow\; C* = P\sqrt{\frac{2bT}{i}}$$

Money Supply - M = $\frac{C*}{2} = \sqrt{\frac{bT}{2i}}$; we can substitute GDP (*Y*) for *T* to get money supply

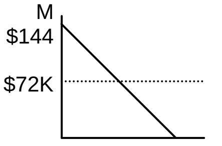
Results - money supply changes in proportion to price level (*P*↑ by 10% ⇒ M↑ by 10%); M increases by less than in proportion to income and decreases by less than in proportion (specifically by $Y^{1/2}$ and $i^{-1/2}$)

Problem - can't really substitute GDP because some transactions require money but aren't captured in GDP (used cars, illegal activities, etc.)

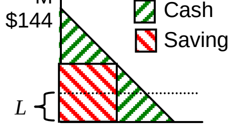
Tobin Model

Scenario - assume *T* = $144K/year; can put money in interest bearing asset that earns *i* = 10%; each transaction costs *b* = $700

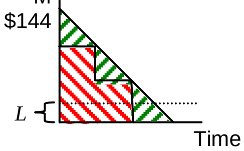
No Bank Trips - demand for money *L* = $144K/2 = $72K; no interest earned and no costs incurred



1 Trip - save 1/2 money at start of year and go back to back to get it in 6 months; *L* = (T/2)/2 = $36K; 2 transactions so cost is 2*$700 = $1400; earn interest on half the money for half the year: *T*/2·*i*·1/2 = $3600



2 Trips - save 2/3 money at start of year; go back to get 1/2 of savings (T/3) in 4 months (1/3 of year); go back again to get rest at 2/3 year; *L* = (T/3)/2 = $24K; transactions cost 3*$700 = $2100; 1/3 of money earns interest for 2/3 year and 1/3 of money earns interest for 1/3 of year: *T*/3·*i*· (1/3+2/3) = $4800



***n* Trips** -

$L = (T/n)/2$

Cost = bn

Interest Earned = $(T/n) \cdot i \cdot n \cdot (n-1)/2$ (Note: sum of t consecutive integers is $t(t+1)/2$)

Decreasing Returns - each additional trip yields less additional interest income

Goal - maximize net benefit (interest earned minus transactions cost); in this scenario, best to make 3 trips so L = $18K

Trips	TX	Cost	Interest	Net	L
0	0	0	0	0	72000
1	2	1400	3600	2200	36000
2	3	2100	4800	2700	24000
3	**4**	**2800**	**5400**	**2600**	**18000**
4	5	3500	5760	2260	14400

Results -

$i\uparrow \Rightarrow$ earnings increase so number of transactions increase... $L\downarrow$

$T\uparrow \Rightarrow L\uparrow$ (regardless of # of transactions); earnings increase so number of transactions increase; <u>manage cash more intensively</u>; hold more money but less than in proportion to ΔT (similar to Baumol model)

Economies of Scale - same cost per transaction so it pays more (on net) to make more transactions when you have more money

Problem - why don't we do this? Get paid twice per month so T very low; doesn't pay to cover transaction cost

Second Problem - getting paid twice per month implies we would hold a week's income in cash which implies V = 52, but it's only 8.8 (we hold 5 weeks income in cash); why?

1. Households vs. firms (firms hold more cash)
2. Most cash is $100 bills... probably held for illegal transactions or overseas
3. Portfolio

Quantity Theory of Money - by Milton Friedman; large amounts of cash are held for portfolio reasons, not just transactions

Empirical Work

Allan Meltzer, "The Demand for Money: Evidence From the Time Series." JPE 1963

Data - annual data from 1900-1950

Logs - use ln(M/P), ln(i), ln(Y), etc. because coefficients can be interpreted as elasticities

Baumol Model - suggests ln(M/P) = [const] - 0.5ln(i) + 0.5ln(Y)

Results -

$\ln(M1/P)$ = [const] - $0.99\ln(i)$ + $1.11\ln(W/P)$ $R^2 = 0.992$ (W = wealth)

(0.04) (0.03)

$\ln(M2/P)$ = [const] - $0.5\ln(i)$ + $1.32\ln(W/P)$ $R^2 = 0.994$

(0.05) (0.02)

$\ln(M1/P)$ = [const] - $0.79\ln(i)$ + $1.05\ln(Y)$ $R^2 = 0.981$

(0.04) (0.04)

$\ln(M1/P)$ = [const] - $0.92\ln(i)$ + $0.97\ln(W/P)$ + $0.13\ln(Y)$ $R^2 = 0.995$

(0.05 (0.10) (0.09) Y not significant

Notes -

1. Makes sense to use M2 with wealth (portfolio) and M1 with income (transactions)
2. W more important than Y for determining money demand

3. Multicollinearity problem; Y & M correlated, but so are W & Y; can't interpret coefficients
4. M/P always gets good fits, but not always good forecasts
5. Coefficient for i is always < 0 and significant; coefficient for scale (W or Y) is always > 0 & significant

Baby Sitting Example

Sweeney & Sweeney, "Monetary Theory and the Great Capitol Hill Baby Sitting Co-op Crisis."

Background - cooperative of 150 families taking advantage of economies of scale in baby-sitting; true cost s sitting there watching "Clifford" so adding another kid doesn't cost much on the margin

Barter - economists usually think of barter as being inefficient because it requires **double coincidence of wants** (have to find someone who wants what you're supplying and has what you want; usually takes along time to find someone to trade with)

Potential Problems - aside from inefficiency of barter, people could abuse the system (use baby sitter 20 times and only sit 5 times)

Solutions -

(1) **bookkeeping** - credit and debit hours of babysitting; problems with accuracy and unpleasant phone calls

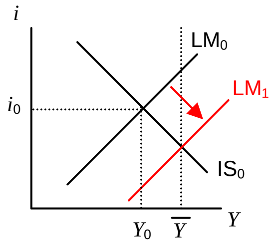
(2) **scrip** - explain rules and give 40 scrip ("money") good for 1/2 hour of babysitting; people use/earn scrip with no bookkeeping required; problems include counterfeiting and people moving (decrease money supply)

$L > M/P \Rightarrow$ people baby sit to earn more scrip

$L < M/P \Rightarrow$ people go out more to get rid of scrip

Crisis - CHBSC used scrip; co-op had administration (people to explain rules, interview new members, etc.); wouldn't work with volunteers so staff was paid with scrip through dues (4200 scrip/year ~ 2100 hours ~ 1hr/mo/couple... call that T = net taxes); problem was staff being too efficient; only used 3800 scrip ~ 1900 hours... call that G = government purchases; money supply was shrinking; result - more people on the list to baby-sit, but fewer people asking for baby sitters (S > D; equivalent of unemployment)

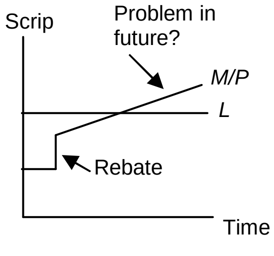
Economist Solution - have administrators redistribute the "surplus" (monetary policy); not obvious to people that money supply is the problem



CHBSC Solution - rule requiring people to go out at least once every six months; most of the members were lawyers and used to "Stalinist, central planning" (Bomberger)

IS-LM - what's needed is either increasing the money supply ($M\uparrow$) or have prices drop ($P\downarrow$); problem is prices are fixed (scrip says "1/2 hour")

Black Market - could have baby sitters offering to work for less ("I'll work for 1 scrip per hour"); could have people going outside the co-op spending real money rather than scrip



Keynesian Idea - monetary is great compared to barter, but has problems when money market is out of balance; need a central bank to maintain $L = M/P$ or need flexible pricing so P can change to maintain balance (problem with sticky prices... especially wages)

Eventual Solution - CHBSC redistributed the "surplus" and gave new members 60 units of scrip (rather than 40); also, members leaving the co-op only have to return 40 units; now have increasing money supply... could cause problems in future

History of Economic Thought on Money

John Maynard Keynes - not first to use money demand or first to talk about recessions; first to use monetary policy to explain recession

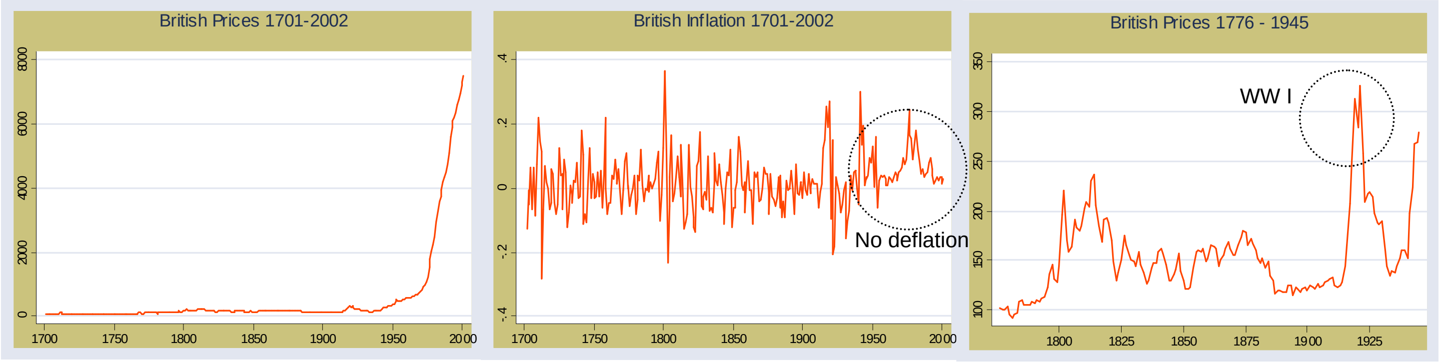
"Common Knowledge" - "downturns are a necessary evil in capitalist economy"; capitalist system is best because it's increases standard of living fastest, but downturns are "necessary" because doesn't seem to be anything to do about it; Keynes was unique because he was the only one not saying recessions would get worse

Minority Opinions -

- **Marx** - downturns are good; capitalist system produces too much then cuts back to get rid of inventories; owners take advantage of workers; said downturns would get worse and worse, but increased dependence of technology would result in educated work force that would eventually overthrow the owners; "Centralization of the means of production and socialization of labor at last reach a point where they become incompatible with their capitalist integument." "Thus integument is burst asunder. The knell of capitalist private property sounds. The expropriators are expropriated."
- **S?????** - government help doesn't let firms eliminate "dead wood" so downturns would get worse over time; “[A recovery] is sound only if it [comes] of itself. For any revival which is merely due to the artificial stimulus leaves part of the work of depressions undone and adds, to an undigested maladjustment, new maladjustment of its own which has to be liquidated in turn, thus threatening business with another [worse] crisis ahead."

Britain - looking at money since 1700s shows not much change until last 50 years, and since then there's been no deflation

- **1776-1900** - price fairly stable because on gold standard; (1) government can't just print money (doesn't have enough gold to back it up); (2) fixes exchange rate between pound and any other currency on gold standard (1 pound = 5 U.S. dollars); designed to prevent persistent inflation



- **WW I** - went off gold standard; tripled money supply which tripled prices
- ***Economic Consequences of the Peace*** - published by Keynes in 1919; said reparations on Germans were shortsighted; just making German democracy unpopular (people would view it as collecting money for the Allies); led to hyperinflation like Keynes predicted
- **After War** - went back to gold standard; had two options: (1) change exchange ratio (i.e., don't reverse the inflation), (2) cut money supply to get back to old exchange ratio; arguments for latter case were to gain credibility for future and to keep financial markets from moving to New York

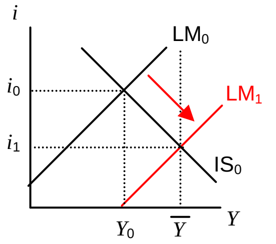
Economic Consequences of Mr. Churchill - published by Keynes in 1925; tried to gain credibility by using similar title to previous paper where he was correct about Germany; argued that going to gold standard at current level would be better

Depression - Britain chose latter policy and had rapid deflation; unemployment was very high; Britain had long depression before the world-wide Great Depression hit in the 30s; people started to argue that Marx was right; Keynes argued it was a monetary problem in his book

A General Theory of Employment, Interest and Money - published by Keynes in 1936; didn't write it as an "I told you so" book because he still wasn't entirely convinced; prices adjusted quickly after Napoleonic (1812) War; Keynes thought deflation would take much longer after WW I because of unions (sticky prices); it happened faster than Keynes thought, but still didn’t have unemployment problem solved

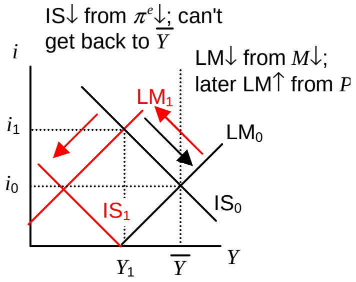
General Idea of Monetary Policy - $P\downarrow \Rightarrow M/P\uparrow \Rightarrow \text{LM}\uparrow \Rightarrow i\downarrow \Rightarrow I\uparrow \Rightarrow Y\uparrow$; key is lower interest rates increasing output via investment

Liquidity Trap - if potential GDP is out too far, monetary policy alone can't solve the problem; eventually get to $i = 0$ from decreasing price level (or increased money supply), but will never get to full employment; once interest rates go to zero, investment is no longer encouraged and money gets "trapped" in portfolios (people hold money rather than invest it)



Pigou Effect - Pigou argued for $C(Y - T, i, \text{Wealth})$; deflation increases wealth which increases consumption; problem is effect of wealth on consumption (and hence output) is weak compared to effect of interest rate on investment (and hence output)

Getting Trapped - $I(i - \pi^e)$; suppose i = 4% and π^e = 2% ∴ real rate = 2%; cut money supply to get to previous exchange rate (move to LM$_1$); wait for deflation to get LM back to full employment, but if it takes too long, people expect deflation so π^e drops (i.e. move to IS$_1$; people expect to pay back loans with money that is worth more so they don't want to borrow); by the time prices fall to original LM curve (i = 4%), still haven't reach potential GDP because real rates are too high



Solution - Keynes argued in this case monetary policy is no good so we need fiscal policy (G↑) to bring economy back; when in a liquidity trap, fiscal policy is fine to use because there is not much crowding out

Keynesian Policy - used as pejorative term for <u>any</u> fiscal policy (usually deficit spending) used to stimulate the economy; **Note:** Keyes argued for monetary policy; only talked about fiscal policy when interest rates were too low for monetary policy to work

Why Study This? - Japan; 1990s look the same; interest rates near zero and high unemployment

US? - target federal funds rate at 1%; lowest in 40 years; been there for a while; if it stays too long, we could have $\pi^e\downarrow$ and end up in liquidity trap

Bomberger - looks like some inflation is good for a monetary policy cushion... maybe 4%

HOPEFUL SIGN

Annual change in Japan's consumer-price index

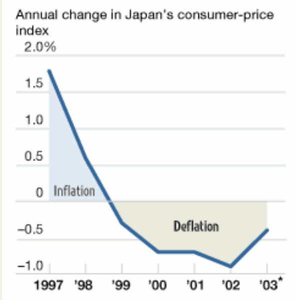
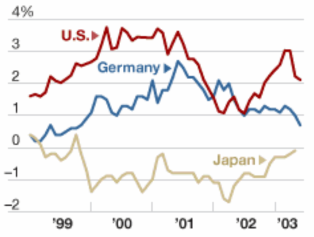


*Figure is from June 2003 vs. June 2002
Source: Japan's Ministry of Public Management, Home Affairs, Posts and Telecommunications

RISKY CYCLE

Prices are starting to fall...
Year-to-year percentage change in consumer prices



LOWER RATES

Sources: Federal Reserve; WSJ Statistics

Missing Money

Goldfield, "The Case of the Missing Money."

Demand for Money (L) - can't really predict it, but we assume it's function of Y and i and that $L_Y > 0$ and $L_i < 0$; assume it's: $\ln(L)_t = a_0 + a_1\ln(Y)_t + a_2\ln(i)_t + \ldots$

Predict L - usually use $L = M/P$ (implicitly assumes market is at equilibrium); Goldfield argued that money demand may adjust slowly because there are two components to the cost of reaching equilibrium:

Partial Adjustment - cost of changing L_t is $a(\ln(M/P)_t - \ln(L)_t)^2 + b(\ln(M/P)_t - \ln(M/P)_{t-1})^2$

Cost of holding non-optimal amount of money | Adjustment cost; cost to change portfolio

Holding Cost - squared because (1) holding too much is just as bad as holding too little, (2) cost grows the farther you are from optimal amount

Adjustment Cost - squared because (1) adjusting cost in both directions, (2) cost of adjustment grows the faster you change

Min Cost - take derivative wrt $\ln(M/P)_t$, set it equal to zero and solve for $\ln(M/P)_t$

d(cost)/d($\ln(M/P)_t$) = $2a(\ln(M/P)_t - \ln(L)_t) + 2b(\ln(M/P)_t - \ln(M/P)_{t-1}) = 0$

$$\ln(M/P)_t = \frac{a}{a+b}\ln(L)_t + \frac{b}{a+b}\ln(M/P)_{t-1} = \lambda\ln(L)_t + (1-\lambda)\ln(M/P)_{t-1}$$

Zero Adjustment Cost - note if $b = 0$, $\ln(M/P)_t - \ln(L)_t$ (i.e., adjust immediately)

Model - substitute in formula for $\ln(L)_t$ and get:

$$\ln(M/P)_t = \lambda a_0 + \lambda a_1\ln(Y)_t + \lambda a_2\ln(i)_t + \ldots + (1-\lambda)\ln(M/P)_{t-1}$$

Rewrite with single parameters (that's what we'll estimate in a regression):

$$\ln(M/P)_t = b_0 + b_1\ln(Y)_t + b_2\ln(i)_t + \ldots + c\ln(M/P)_{t-1}$$

Interpreting Coefficients - note that $1 - \lambda = c$ (or $\lambda = 1 - c$); now $b_1 = \lambda a_1 \Rightarrow a_1 = b_1/(1-c)$; similarly $a_2 = b_2/(1-c)$

"Short-Run" Elasticities - b_1, b_2, etc. (1 quarter)

"Long-Run" Elasticities - a_1, a_2, etc. (eventual)

Goldfield's Results - quarterly data 1952:2 to 1973:4

$$\ln(\mathrm{M1}/P)_t = [\text{const}] + 0.179\ln(Y)_t - 0.042\ln(i_{\mathrm{td}})_t - 0.181\ln(i_{\mathrm{cp}})_t + 0.676\ln(\mathrm{M1}/P)_{t-1}$$

$R^2 = 0.995$ (0.04) (0.011) (0.003) (0.068)

$$\ln(\mathrm{M2}/P)_t = [\text{const}] + 0.206\ln(Y)_t - 0.021\ln(i_{\mathrm{tb}})_t - 0.071\ln(i_{\mathrm{sl}})_t - 0.029\ln(i_{\mathrm{td}})_t + 0.884\ln(\mathrm{M2}/P)$$

$R^2 = 0.999$ (0.054) (0.003) (0.028) (0.012) (0.045)

Problem - good fit doesn't imply good forecast; forecast error for 1976:2: -22% for M1 and -4.4% for M2

Reason - overestimated real money balances because he underestimated inflation; problem wasn't too little money, but too much inflation

Example - $\mu_i = (d\mathrm{M}i/dT)/\mathrm{M}i$ = rate of growth of Mi (i = 1 or 2); π_i = inflation rate predicted implicitly by Goldfield's model; π = actual inflation rate

	μ_1	μ_2	π_1	π_2	π
1973	4.9%	5.7%	5.2%	5.5%	9.1%
1974	4.8%	5.8%	5.5%	6.7%	11.4%
1975	5.2%	11.4%	3.7%	7.3%	7.1%

Lesson - overestimating real money balances is equivalent to underestimating inflation; problem wasn't missing money, but too much inflation

The Federal Reserve

Required Reserve Ratio - amount of money banks are required to hold in cash at the end of each day as percentage of total deposits

Federal Funds Rate - interest rate banks charge each other to borrow money (usually lent daily to cover RRR requirement)

Open Market Operations - buy/sell government securities in order to achieve monetary policy objectives (manipulate M or FFR)

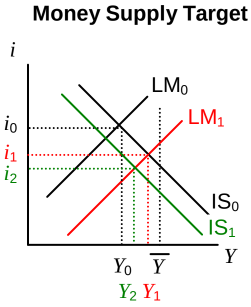
- **Expansionary** - buy bonds ⇒ M↑ ⇒ banks have more cash (excess reserves) ⇒ FFR↓
- **Contractionary** - sell bonds ⇒ M↓ ⇒ banks have less cash (harder to make RRR) ⇒ FFR↑

Money Supply Target - Fed focused on (dM/dt)/M (growth rate of money supply) and set targets for M1, M2, and M3 growth; usually only concerned about a range for FFR (used in 80s)

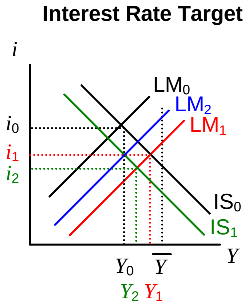
Interest Rate Target - Fed sets very narrow range for FFR and doesn't worry about money supply (current policy)

Potential Problems - unforeseen problems cause problems with monetary policy; say Fed targets specific ΔM or wants FFR = i_1 to get to Y_1 (and i_1)

(1) IS↓ (for any reason, but usually π^e↓ or I↓)

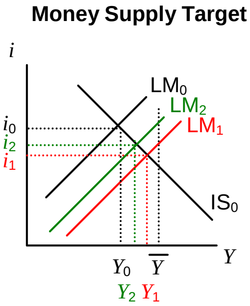


IS↓ keeps Fed from target; i & Y lower

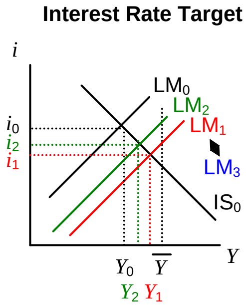


IS↓ makes Fed lower ΔM to keep on I target; actual Y even lower than with money supply target

(2) LM↓ (from L↑)



L↑ (LM↓) cancels part of Fed's ΔM; i higher & Y lower than target



L↑ (LM↓) has no effect because Fed focused on i; gets to target (takes care of "missing money" problem)

Summary - if IS is more unpredictable, use money supply target because interest rate target exaggerates ΔY from ΔIS (that's bad... recessions worse); if L (LM) is more unpredictable, use interest rate target because money supply target is countered by ΔY from ΔLM (policy is ineffective... not as bad as the exaggeration problem with IS)

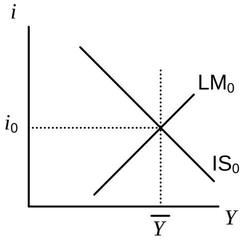
Real World - usually (at least currently) L is more difficult to predict so Fed focuses on target FFR

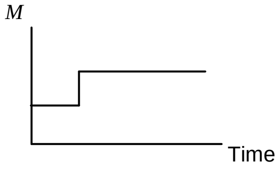
Inflation and Unemployment

No Growth Model - IS-LM model we looked at before

$\frac{dP}{dM} = \frac{M}{P} > 0$ (Price changes in proportion to M)

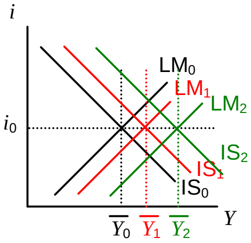
$\frac{dY}{dM} = \frac{di}{dM} = 0$ (No effect on Y or i in long run)





Growth Model - assumes output is growing over time; we'll look at specific case where production function is Cobb-Douglas and %ΔIS = %ΔLM ⇒ constant interest rate

Purpose - we want to be able to look at changing the growth rate of the money supply ($\mu = dM/dT)/M$), not just a one time change in the money supply (M)



Equations -

Static	Specific/Growing
$Y = F(K, \overline{N})$	$Y = AK^{\alpha}\overline{N}^{1-\alpha}$
$W/P = F_N(K, N)$	$W/P = (1-\alpha)AK^{\alpha}N^{-\alpha}$
$N = \overline{N}$	$N = \overline{N}$
$C = C(Y - T)$	$C = c(Y - T)$
T exogenous	$T = tY$
G exogenous	$G = g\overline{Y}$
$I = I(i - \pi^e)$	$I = h(i - \pi^e)\overline{Y}$
$M/P = L(i,Y)$	$M/P = l(i)Y$

Before consumption was generic function C of net income (Y - T); now it's a specific function c times (Y - T); c = marginal propensity to consume

$$\left.\begin{matrix} C = c(Y-T) \\ T = tY \end{matrix}\right\} \rightarrow C = c(1-t)Y$$

Taxes (T) change proportionately with actual output Y

Gov't purchases (G) change proportionately with potential output $\overline{Y}$

Investment (I) changes proportionately with potential output $\overline{Y}$; $h' < 0$

Money demand (L) changes proportionately with output Y; $l' < 0$

Math Tricks -

%Δ(M/P) = %ΔM/M - %ΔP/P

%ΔM/M = (dM/dT)/M = μ (here T stands for time, not taxes)

%ΔP/P = (dP/dT)/P = π

%Δ($l(i)Y$) = %Δl/l + %ΔY/Y

%Δl/l = 0 (because we're assuming i doesn't change)

%ΔY/Y = (dY/dY)/Y = y

$\therefore y = \mu - \pi$ (rate of growth equals money growth rate minus inflation rate)

Result - rearrange terms: $\pi = \mu - y$ to get formal relationship between π and μ shown below (crosses μ axis at positive μ and has slope of 1)

Changing Variables - look at changes in growth model

Static	Specific/Growing
M fixed	$(dM/dT)/M = \mu$
$\overline{N}$ fixed	$(d\overline{N}/dT)/\overline{N} = n$
K fixed	$(dK/dT)/K = k$
A fixed	$(dA/dT)/A = a$
$\overline{N}$ fixed	$(d\overline{N}/dT)/\overline{N} = n$

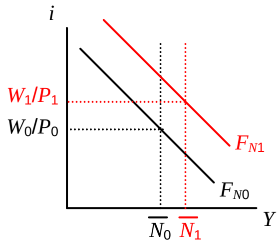
Not the same k from Solow model (output per worker); now k = growth rate of capital

Output Growth - get a result similar to Solow model... output growth depends on technology, capital, and labor growth rates

$$y = \frac{dY/dT}{Y} = \frac{d(AK^{\alpha}\overline{N}^{1-\alpha})/dT}{AK^{\alpha}\overline{N}^{1-\alpha}} =$$

$$\frac{K^{\alpha}\overline{N}^{1-\alpha}(dA/dT) + \alpha AK^{\alpha-1}\overline{N}^{1-\alpha}(dK/dT) + (1-\alpha)AK^{\alpha}\overline{N}^{-\alpha}(d\overline{N}/dT)}{AK^{\alpha}\overline{N}^{1-\alpha}} =$$

$$\frac{dA/dT}{A} + \alpha\frac{dK/dT}{K} + (1-\alpha)\frac{d\overline{N}/dT}{\overline{N}} = a + \alpha k + (1-\alpha)n$$



Note: μ is not in this equation

Labor Growth - confirms another result of the Solow model

$\%\Delta(W/P) = \%\Delta W/W - \%\Delta P/P = w - \pi$

$\%\Delta W/W = (dW/dT)/W = w$ (% change in nominal wages)

$\%\Delta P/P = (dP/dT)/P = \pi$ (inflation rate)

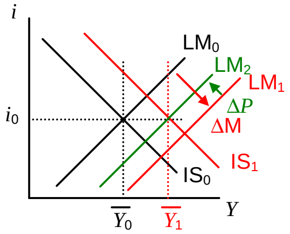
$$\%\Delta(W/P) = \frac{d(W/P)/dT}{W/P} = \frac{d((1-\alpha)AK^{\alpha}N^{-\alpha})/dT}{(1-\alpha)AK^{\alpha}N^{-\alpha}} =$$

$$\frac{K^{\alpha}N^{-\alpha}(dA/dT) + \alpha AK^{\alpha-1}N^{-\alpha}(dK/dT) - \alpha AK^{\alpha}N^{-\alpha-1}(dN/dT)}{AK^{\alpha}N^{-\alpha}} =$$

$$\frac{(dA/dT)}{A} + \alpha\frac{(dK/dT)}{K} - \alpha\frac{(dN/dT)}{N} = a + \alpha k - \alpha n$$

Result - $\%\Delta(W/P)$ (% change in real wages) = $w - \pi = a + \alpha k - \alpha n = y - n$ (substituting equation y we found above); also, using the math trick above with %Δ of ratios, $\%\Delta(Y/N)$ [i.e., output per worker] = $y - n$; that confirms the Solow result: the growth rate of real wages = the growth rate of worker productivity

Money Growth - suppose constant money growth μ; this means LM curve steadily shifts right



Inflation - if $\mu > \%\Delta$IS, there will be inflation

Example - $\mu = 6\%$ & $\%\Delta\text{IS} = 3\% \Rightarrow \pi = 3\%$

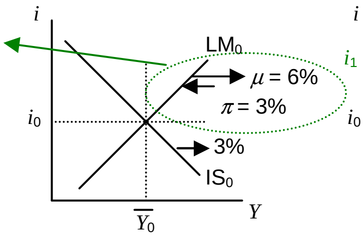
Stationary Graph - assume we're panning so IS-LM curves don't shift; just have arrows showing direction of change; if we print money faster (e.g., $\mu\uparrow$ from 6% to 10%), result is growth in inflation ($\pi\uparrow$ from 3% to 7%... amount is same: 4%)

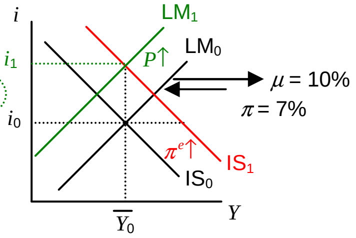
$\Delta\pi$ causes π^e to change too (↑), which effectively lowers real interest rate ($i_0 - \pi^e$)... shifts IS right

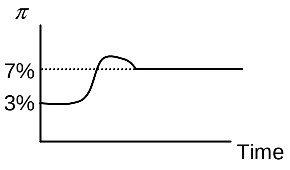
IS↑ countered by $P\uparrow$ which shifts LM left so $i\uparrow$

Prices increase faster than inflation this year, then settle down so $\Delta i = \Delta\pi^e = \Delta\pi = \Delta\mu$

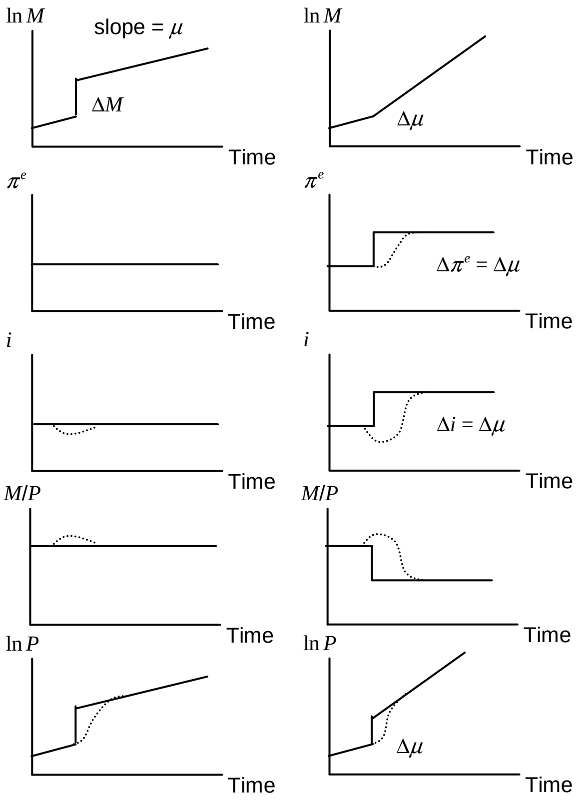
Note: this illustrates the equation we had earlier: $y = \mu - \pi$







Increase *M* vs. Increase μ



Monetary Neutrality - no change in real variables (Y, C, I, i, W/P, M/P) in long run from ΔM

Monetary Superneutrality - no change in real variables (Y, C, I, i, W/P, M/P) in long run from $\Delta\mu$; doesn't hold because $d(M/P)/d\mu \neq 0$

Graphs - solid lines show instantaneous changes; dashed lines show more realistic changes over time

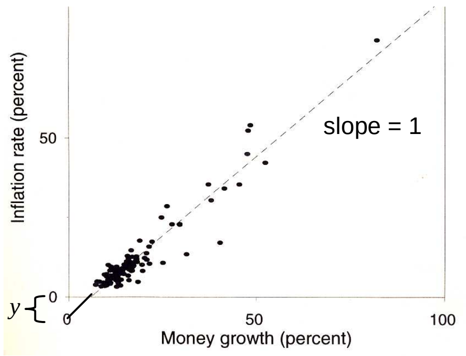
	ΔM	$\Delta\mu$
Y	$dY/dM = 0$	$dY/d\mu = 0$
C	$dC/dM = 0$	$dC/d\mu = 0$
I	$dI/dM = 0$	$dI/d\mu = 0$
i	$di/dM = 0$	$di/d\mu = 1$
M/P	$d(M/P)/dM = 0$	$d(M/P)/d\mu = l'Y < 0$
P	$dP/dM = P/M$	$dP/d\mu = -P\,l'Y/l > 0$
π	$d\pi/dM = 0$	$d\pi/d\mu = 1$

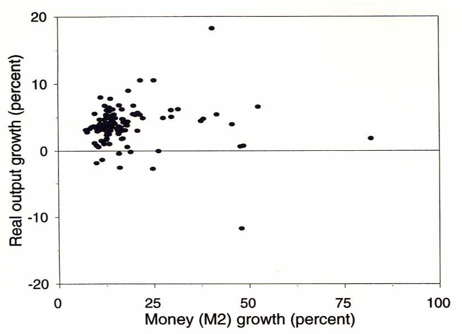
ΔM - change money supply; affects P, but not Y or i (real variables); monetary neutrality (realistic)

$\Delta\mu$ - change rate of growth; affects π, but not Y or i

π vs. μ - notice intercept isn't at (0,0); that's because there's real growth so you can have increasing money supply (μ) without inflation (π); we found the formal relationship earlier: $\pi = \mu - y$

y vs. μ - note that there doesn't appear to be a relationship between money growth rate and real output growth





Phillip's Curve

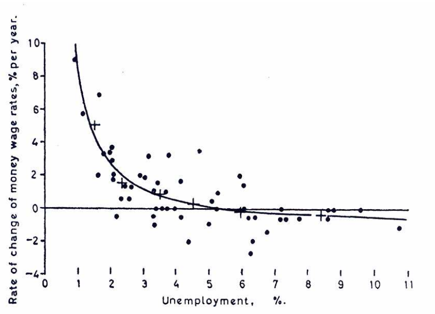


Fig.1. 1861 – 1913

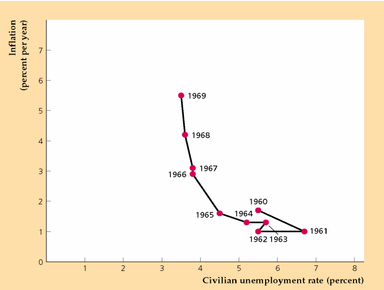
Phillip's Curve - plotted rate of change of money wage rates vs. unemployment (data from Brittan, 1861 to 1913) and concluded that there's a trade off between inflation and unemployment; higher inflation means lower unemployment

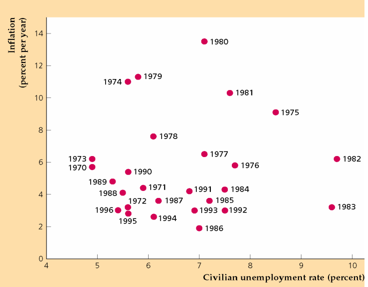
Why Care - Okin's Law says for each % drop in unemployment rate, output (Y) increases by 2%; Phillips curve looks like we can trade off higher inflation in order to get lower unemployment

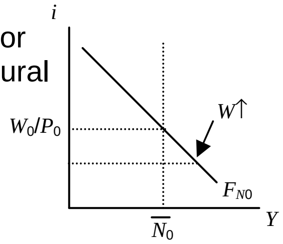
Influential - suggests increasing money growth rate in order to causes inflation and lower unemployment

Realistic? - data fit very well for U.S. in 60s, but not afterwards

Theory - if wages are increasing ($W\uparrow$), it appears we started above $\overline{N}$; that suggests we can have $w > 0$ or $U > 0$, but not both... not realistic because of structural unemployment







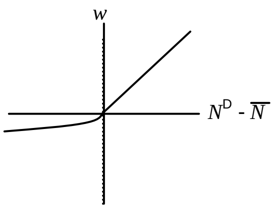
Structural Unemployment

Adding structure to labor market can result in unemployment even when total demand for labor equals total supply ($N^D = N^S$)

Numerical Example - assume 4 different labor markets, each with supply of 100 workers (total supply of labor = 400 workers); with single labor market, as soon as demand for labor equals 400, unemployment goes to zero; with structured markets, it's possible to have demand exceed supply and still have unemployment (workers not in right market)

A	B	C	D	Overall			Single Market		Structured Market	
N^D	N^D	N^D	N^D	N^D	N^S	w	U	u	U	u
70	80	90	100	340	400	-6%	60	15%	60	15%
75	85	95	105	360	400	-4%	40	10%	45	11%
80	90	100	110	380	400	-2%	20	5%	30	8%
85	95	105	115	400	400	0%	0	0%	20	5%
90	100	110	120	420	400	2%	0	0%	10	3%
95	105	115	125	440	400	4%	0	0%	5	1%
100	110	120	130	460	400	6%	0	0%	0	0%

Wage Stickiness - $w = \beta\left(\dfrac{N^D - \overline{N}}{\overline{N}}\right)$



β is measure of how responsive wages are; higher value means more responsive; table above uses $\beta = 0.4$

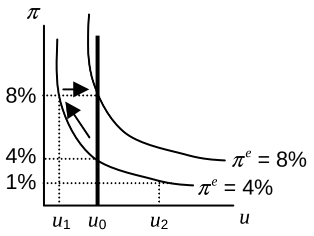
Downward Stickiness - graph shows stickiness for wages (they don't go down much); has different β when N^D - $\overline{N}$ is negative vs. positive

Non-linearity of Phillips Curve - caused by different labor markets and sticky wages

Saving Phillips Curve - Friedman article ("The Role of Monetary Policy," 1968) explained why Phillips curve does fit U.S data

$w = \beta\left(\frac{N^{\mathrm{D}} - \overline{N}}{\overline{N}}\right) + \pi^e$ ... usually have inflation (π) = wage growth rate (w) so changes in unemployment rate $(N^{\mathrm{D}} - \overline{N})\overline{N}$ result from disparity between π and π^e

Sticky Wages - usually wages determined by contracts which try to anticipate w by estimating $N^{\mathrm{D}} - \overline{N}$ and π (using π^e)



Short-Run Phillips Curve - plots unexpected π (i.e., $\pi - \pi^e$) vs. u

$\pi^e < \pi \Rightarrow w$ lower in real terms so $N^{\mathrm{D}} > \overline{N}$ (tight labor market) $u\downarrow$

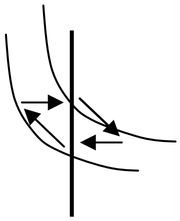
$\pi^e > \pi \Rightarrow w$ higher in real terms so $N^{\mathrm{D}} < \overline{N}$ (excess supply) $u\uparrow$

Long-Run Phillips Curve - vertical line at natural rate of unemployment

Natural Rate of Unemployment - natural doesn't necessarily mean unchanging, just not influenced by M or μ; changes based on mismatches in structure of labor markets

Goes Down - retrain/move workers; wages adjust

Goes Up - price shocks (e.g., oil crisis in 70s); technology (changes increase mismatches in labor markets; e.g., programmers vs. typists)



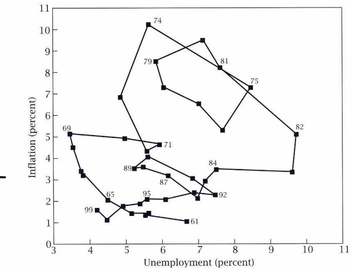


FIGURE 5.17 Unemployment and inflation in the United States, 1961–1999

Loops - if π keeps going up and down, expect clockwise loops, just like we see in U.S. data

Conclusion - can't use monetary policy to influence unemployment in long-run

Original Phillips Curve - very stable because Britain was on the gold standard so π^e was very stable (single curve)

Illusion - since there was only a single curve, the original paper by Phillips seemed to suggest we could permanently stay at high π and low u (which Friedman and U.S. data argued we can't)

Changes in Natural Rate - study by Lilien trying to measure impact of shocks on unemployment (1982)

$$u_t = \underset{(11.8)}{52.6}\sigma_t - \underset{(9.3)}{15.3}(\pi_t - \pi_t^e) - \underset{(10.1)}{16.6}(\pi_{t-1} - \pi_{t-1}^e) + \underset{(0.114)}{0.728}u_{t-1} \qquad R^2 = 0.739$$

σ_t = standard deviation of employment in 11 broadest classifications of industry; evidence of shock when σ_t is big

Negative Coefficients - make sense because actual inflation > expected inflation means unemployment goes down

Higher Unemployment in Europe - various suggestions to explain it

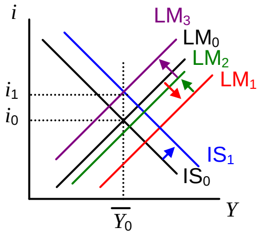
- regulation in labor market
- high unemployment benefits
- reluctant to move (more mismatch in labor markets)

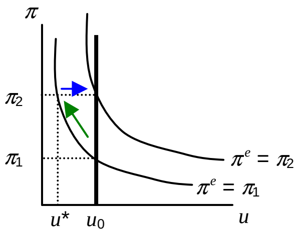
Role of Monetary Policy - in addition to fixing the Phillips Curve, Friedman's article ("The Role of Monetary Policy") talked about the proper role; looked at 3 cases:

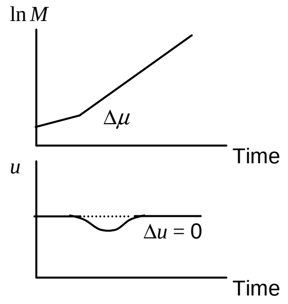
"Peg" u - try to keep unemployment rate (u) at u* by printing money faster (or slower)

Events - μ↑ ⇒ LM↑ ⇒ LM↓ (from P↑, but not all the way back to original level if you believe in Phillips Curve [trading π for u]) and IS↑ (from π^e↑ [i.e., I↑]) ⇒ LM↓ (from additional P↑ or W↓)

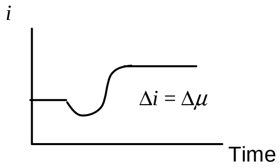
Results - $\Delta\pi = \Delta\pi^e = \Delta i = \Delta\mu$ ⇒ real interest rates (r) don't change; works in short run (temporary fix), but eventually return to u_0 (natural rate or "full" employment); to stay at u*, need to keep increasing M resulting in accelerating inflation (not the same as Phillips' original conclusion of a one-time trade off between π and u)







"Peg" i - try to keep interest rate (i) at i* by printing money faster (or slower)



Events - same as trying to peg u; note that i drops initially, but ends up with $i_1 > i_0$

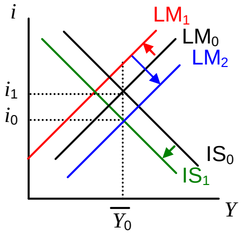
Results - worse than trying to peg u; not only does it lead to accelerating inflation, but it ends up driving interest rates the wrong way

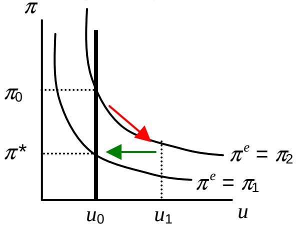
Real World - Fed doesn't really target interest rate in and of itself; interest rate targets are a short run way of getting economy to full employment; that's why Fed is always changing the target interest rate

"Peg" π (or μ) - try to keep inflation under control to π*

Events - assume π too high; want to get it down so μ↓ ⇒ LM↓ ⇒ IS↓ (from π^e↓ [i.e., I↓]) ⇒ LM↑ (from P↓; higher real money balances); this is exact opposite of IS-LM used for peg u above (that would be the case of wanting to increase inflation)

Results - achieving specified inflation rate is feasible; Fed doesn't target inflation because it's necessarily most important; it's just the only thing that can be controlled; **Note:** trying to bring inflation down causes a temporary increase in unemployment

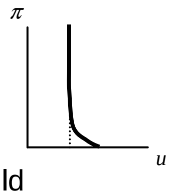




Note: If Fed want to increase of decrease money supply (M), usually equivalent to increase of decrease rate of growth of money supply (μ) so "decrease" doesn't necessarily mean less money in the economy

Why not peg π at zero? (or even negative?)

- **Non-Vertical Phillips Curve** - Tobin, "Inflation and Unemployment"; argued that long-run Phillips curve is not vertical because wages are sticky downward; ∴ as $i\downarrow$ below a certain level, $u\uparrow$
  - **Advantage of π** - inflation adds flexibility to the labor market; allows wages to adjust faster (people don't like a cut in nominal wages, but not adjusting for inflation is equivalent to a decrease in real wages)
- **Liquidity Trap** - if there's a downturn from a change in aggregate demand (i.e., IS↓), could get to $i = 0$ so there's no room to use monetary policy



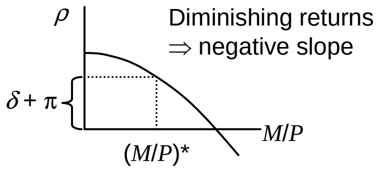
Where to Set Inflation - we know we can't purposely set inflation higher to improve unemployment as Phillips proposed; we also saw that we don't want inflation at zero; what's the right level? is there a benefit from high or low inflation? That's what we'll cover in the next section

Monetary Policy and Stabilization

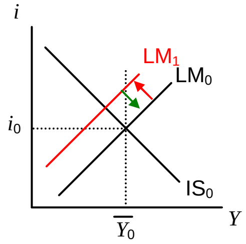
Background - Phillips curve suggests (incorrectly) we can trade higher π for lower u; Okin's Law gives us the benefit (increase Y by 2% for each 1% less u); only thing we know about cost of higher π is decreased real money balances (M/P); need to compare gains (ΔY) with cost ($\Delta(M/P)$)

Optimum Quantity of Money - article by Friedman (1969); real title should be optimal inflation rate; looked at costs and benefits for higher or lower inflation rate to determine the best rate

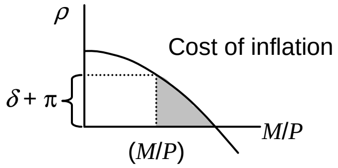
- **Assumption** - initial assumption to make numbers easier is that there is no growth, no inflation, and no securities (i.e., money is the only asset)
- **Benefits of Holding Money** - ρ measures rate of return on services of money
  - **Pecuniary Services** - "shoe leather costs"; higher money balance yields return (less time getting it out of the vault; could use it to produce something)
  - **Non-Pecuniary Service** - "utility"; holding more money allows person to withstand bad times (emergency spending) without diminishing lifestyle
  - **Diminishing Returns** - $(M/P)\uparrow \Rightarrow \rho\downarrow$



- **Cost of Holding Money** - consume less now (don't worry about interest because we assumed money is the only asset and there's no inflation); δ is discount rate of consumption (measure of impatience, how hard it is to hold money to next period); will be different for each consumer, but can look at is as average in aggregate
  - **Adding Inflation** - increases cost of holding money so cost is now $\delta + \pi$
- **Private Optimal** - with no inflation, should have $\rho = \delta$; adding inflation means $\rho = \delta + \pi$
- **Social Optimal** - Friedman argued that there is a positive externality to money holding; if people want to hold more money they do so at no social cost; first they postpone or reduce consumption to increase their demand for money ($L\uparrow \Rightarrow$ LM↓), eventually prices will fall increasing real money balances ($P\downarrow \Rightarrow$ LM↑); problem occurs if only one person does this; everyone else benefits from the higher M/P, but the individual suffers less consumption without making it up completely; since there is zero social cost, the optimal amount of money occurs at $\rho = 0 \Rightarrow \pi = -\delta$ (Friedman was saying the government should induce negative inflation to subsidize money holding)



  - **Being Away from Optimal** - cost of being away from optimal M/P is area under the curve



- **Numerical Example** - using Baumol model (transactions demand for money... **note:** this will overestimate the cost of inflation): $L = L_0 Y^{1/2} i^{-1/2}$ which allows us to calculate money demand, L, which we substitute for real money balances M/P; given current M/P = \$1200B, i = 5%, π = 2%, we know r = i - π = 3%; this allows us to find i for different levels of π, which then lets us calculate (M/P)t = 1200/sqrt(it/5%); i is the benefit of holding money so ρ = i (for this example); create table and find δ by using π = 0 (looking for ρ = δ so in this case, δ = 3%); the cost of inflation is the area under the curve, which equals i(M/P); not that the current situation (π = 2%) costs \$60B... in an economy with Y = \$10000B, this is less than 1 percent so <u>cost of higher in inflation is fairly low; much lower than the gains that resulting from lower unemployment</u>

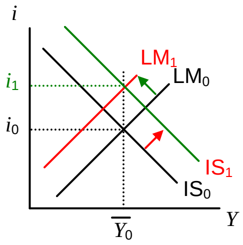
π	i	M/P	ρ	**Cost =** $i(M/P)$
-2%	1%	2683	1%	27
0	3%	1549	3%	46
2%	5%	1200	5%	60
4%	7%	1014	7%	71
6%	9%	894	9%	80
8%	11%	809	11%	89
10%	13%	744	13%	97

Additional Benefit - higher inflation also forces higher I (if constant r) so it helps prevent a liquidity trap, so having a modest amount (2-4%) is good

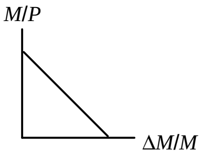
Real World - the fact that economies with 50% inflation function suggests costs of high inflation aren't so great when compared to having 50% unemployment, but there can be too much inflation...

Hyperinflation (Inflationary Finance)

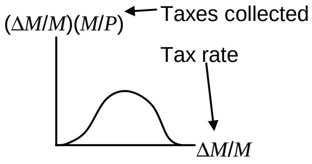
Cagan - "monetary dynamics of hyperinflation"; studied classic example of hyperinflation (post-World War I Germany; unpopular government couldn't increase taxes; it borrowed until creditors wouldn't lend anymore; then it printed money to cover spending)
- **Inflationary Finance** - print currency to finance government purchases; very inflationary
  - **U.S. System** - Federal Reserve's assets include bonds; liabilities are cash in circulation; inflationary finance would involve government issuing new bonds for $xB and the Fed then buying $xB in bonds (by printing money)
- **Example** - assume government wants to purchase some real amount of goods each year
  - **Borrowing** - $G\uparrow \Rightarrow$ IS$\uparrow \Rightarrow W\uparrow \Rightarrow$ LM$\downarrow$; end result is $i\uparrow$, but it's a 1 time Δi and it's not that significant based on real world experience (e.g., $400B deficit right now and $\pi = 2\%$)



  - **Printing** - running the numbers shows very high inflation in first period (32%) and then inflation stays at 10% every year after; considering $60B is such a small % of GDP, it seems odd that inflation would be so high
    - **Money-Balance Tax** - effect of inflationary finance is basically a tax on money-balances; people essentially have less money because of inflation;
      - **Money Raised** - $\Delta M/P$ = effective amount of money raised by printing ΔM
      - **Tax** - multiply $\Delta M/P$ by M/M to get $(\Delta M/M)(M/P)$; the "inflation tax rate is $\Delta M/M$ and the "tax base" (i.e., real money balances) is M/P
      - **Good** - no forms to fill out; people can't evade it
      - **Bad** - money balances are low (compared to income tax based on Y) so the "tax" (i.e., inflation) has to be high; people can avoid it by not holding cash (which makes money balances even lower, raising inflation, causing more avoidance... etc.)
        - **Avoidance** - people can't evade the tax if they hold money, but they can avoid it by not holding money; if $\Delta M/M\uparrow$, then $M/P\downarrow$



    - **Maximize Tax Revenue** - in order to find maximum of Laffer Curve, we need to know the demand for money (M/P)



Demand for Money - Cagan figured out demand for money for Germany

Functional Form - $\ln(M/P)_t = \alpha_0 - \alpha_1 i_t$ this wouldn't work in Germany because credit markets collapsed; the opportunity cost of money was the decrease in value (i.e., inflation), ∴ Cagan used $\ln(M/P)_t = \alpha_0 - \alpha_1 \pi_t^e$

Adaptive Expectations - model for estimating expected inflation: $\pi_t^e = \beta\pi_t + (1 - \beta)\pi_{t-1}^e$ so this period's expectation of next period's inflation (π_t^e) is a weighted average of this period's inflation (π_t) and last period's expectation of this period's inflation (π_{t-1}^e)

Results - Cagan computed for various values of β (0, 0.1, 0.2, etc... no computers back then); best fit was $\beta = 0.2$ resulting in $\alpha = 5.46$ ($R^2 = 0.992$)

Max Tax Revenue - revenue = $\pi \cdot M/P = \pi(m_0 e^{-\alpha\pi}) \Rightarrow \pi^* = 1/\alpha = 0.183$ (per month!) with max revenue of 5.7M; note: Germany was trying to raise more than this at 20% inflation and increased to 40%... revenue dropped from 3.1M to 1.9M; they should've printed less money to get inflation down and real revenue from printing less money would've been more

More Phillips Curves

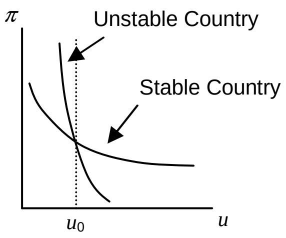
(Trying to explain effect of unexpected inflation)

Lucas - "Some International Evidence on Output Inflation Tradeoffs"; tried to explain why some countries experience swings in prices (π) and other in output (Y) in response to demand shocks (Δx)

Aggregate Demand Shock - demand changes for all goods (up or down); people spend less (or more) on everything... tend to be temporary

Relative Demand Shock - people aren't spending less or more overall, but less or more in a specific industry (e.g., less beer and more wine)... tend to be permanent

Problem - Lucas argued that in the short-run, firms can't tell the difference between an aggregate demand shock and a relative demand shock; their response to a demand shock depends on the country's stability; more stable economies (i.e., more predictable Y) tend to mistake aggregate demand shocks for relative demand shocks; they change output first so shock is evidenced in output (which affects unemployment); firm's in countries were Y is unpredictable uses prices first to deal with shocks



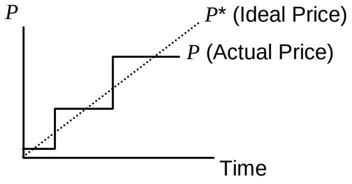
Result - stable economies have flatter Phillips Curve (i.e., slopes different)... this should be testable which is what Lucas did

Model - Y_t = constant + $\alpha\Delta x_t + \beta Y_{t-1}$ (Δx is demand shock)

U.S. - $\alpha = 0.91$... 91% of demand shock goes to increased output

Argentina - $\alpha = 0.01$.. only 1% of demand shock goes to output

Ball, Mankiw & Romer - "The New Keynesian Economics and the Output-Inflation Tradeoff"; alternative explanation for swings in prices vs. output

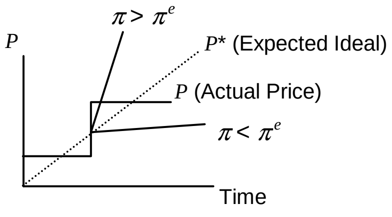


Menu Costs - costly for firms to change prices so firms respond slowly to shocks; they rather change output than prices so inflation looks more like a step function

Actual Prices - tend to fall below ideal prices, then firms adjust by looking at π^e; they overshoot because they know they won't adjust prices again for a while

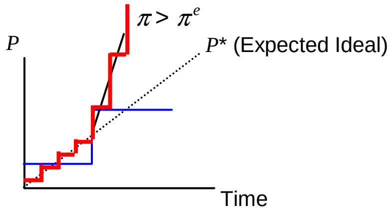
Unexpected Inflation - if $\pi \neq \pi^e$ there are problems

$\pi > \pi^e$ - prices higher than anticipated so firm is undercharging; sales skyrocket and output increases; firm has to hire more workers ($u\downarrow$)



$\pi < \pi^e$ - prices lower than anticipated so firm is overcharging; sales plummet and output falls; firm has to fire workers ($u\uparrow$)

Note: This is same conclusion as Fischer with labor contracts, just different reasoning



"Cheaper" Menu Costs - firms would change prices more often; in that case firm adjusts prices quicker to P adjusts rather than u... i.e., don't deviate from u as much and return quicker... that means Phillips Curve is steeper

Testing Theory - collected time-series data to get money demand:

	σ	π	α
Germany	0.02	0.04	0.61
Italy	0.06	0.08	0.20
Argentina	0.42	0.54	-0.005

Notes: σ = standard deviation of Δx (demand shock); α = coefficient for money demand (similar to Cagan's model)

Cross-Section - next used cross-section of data to look at relationship of α (Lucas expected $\sigma\uparrow \Rightarrow \alpha\downarrow$

Version 1 - $\alpha = \text{constant} - \underset{(1.5)}{4.2\sigma} + \underset{(4.1)}{7.5\sigma^2}$ $R^2 = 0.24$

Version 2 - $\alpha = \text{constant} - \underset{(1.1)}{4.2\pi} + \underset{(2.1)}{7.1\pi^2}$ $R^2 = 0.34$

Result - could argue second version is better (i.e., inflation is better fit than unpredictable demand... somewhat supports menu costs (Ball, Mankiw, Romer) over shock theory (Lucas)

Review

Topics & Authors

Final exam is <u>not</u> cumulative, but still need to know IS-LM (no multipliers)

- **Shifts in IS** - IS↑ (i.e., curve shifts to the right) if $T\downarrow$ ($C\uparrow$), or $\pi_e\uparrow$ ($I\uparrow$), or $G\uparrow$; results in larger output (Y) for given interest rate (i)
- **Shifts in LM** - LM↑ (i.e., curve shifts to the right) if $M\uparrow$, $P\downarrow$, or L↓; results in larger output (Y) for given interest rate (i)

Investment

- <u>Modigliani & Miller</u> - "The Cost of Capital, Corporation Finance and the Theory of Investment"; if firms have same expected stream of income and same variance (risk), then market value of equity plus debt is constant (e.g., firm 1 has no debt and firm 2 has debt D_2: $V_1 = V_2 + D_2$)

Time Series Models

Money Demand

- <u>Baumol</u> - "The Transactions Demand for Cash: An Inventory Theoretic Approach"; based on transaction demand for money; looks at keeping money like stocking inventory; short on intuition, but empirically testable
- <u>Tobin</u> - "The Interest Elasticity of the Transactions Demand for Cash"; similar to Baumol's model, but more intuitive (didn't come up with specific equation)
- <u>Meltzer</u> - "The Demand for Money: Evidence From the Time Series"; empirical work trying to estimate parameters from Baumol & Tobin's money demand models
- <u>Sweeney & Sweeney</u> - "Monetary Theory and the Great Capitol Hill Baby Sitting Co-op Crisis"; administration's budget surplus took "money" (scrip) out of circulation; solution was to have administrators redistribute the "surplus" (monetary policy)
- <u>Goldfield</u> - "The Case of the Missing Money"; Goldfield argued that money demand may adjust slowly because there are two components to the cost of reaching equilibrium: holding cost (for not holding proper amount) and adjustment cost (to get to proper amount); end result: overestimating real money balances is equivalent to underestimating inflation; problem wasn't missing money, but too much inflation

Inflation & Unemployment

- <u>Phillips</u> - "The Relationship Between Unemployment and the Rate of Change of Money Wages in the U.K. 1861-1957"; plotted rate of change of money wage rates vs. unemployment (data from Brittan, 1861 to 1913) and concluded that there's a trade off between inflation and unemployment; higher inflation means lower unemployment
- <u>Friedman</u> - "The Role of Monetary Policy"; explained why Phillips curve <u>does</u> fit U.S data; came up with long-run Phillip's Curve; multiple short-run curves each at different π^e; equilibrium at natural rate of unemployment; only time we trade off π for u is when it's unexpected ($\pi \neq \pi^e$); also examined 3 possible roles for monetary policy: "pegging" u, I, or π... concluded that π is the only thing we can control in long-run with monetary policy
- <u>Lilien</u> - "Sectoral Shifts and Cyclical Unemployment"; empirical work trying to measure impact of shocks on unemployment;
- <u>Tobin</u> - "Inflation and Unemployment"; argued that long-run Phillips curve is not vertical because wages are sticky downward; $\therefore$ as $i\downarrow$ below a certain level, $u\uparrow$

Monetary Policy and Stabilization

<u>Friedman</u> - "The Optimum Quantity of Money"; looked at costs and benefits for higher or lower inflation rate to determine the best rate; also said there's a positive externality to money holding; government should induce negative inflation to subsidize money holding

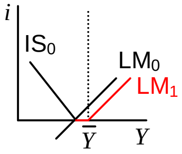
<u>Cagan</u> - "The Monetary Dynamics of Hyperinflation"; studied classic example of hyperinflation; post-World War I Germany; unpopular government couldn't increase taxes; it borrowed until creditors wouldn't lend anymore; then it printed money to cover spending

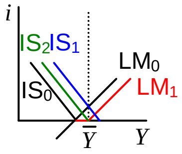
<u>Lucas</u> - "Some International Evidence on Output Inflation Tradeoffs"; tried to explain why some countries experience swings in prices (π) and other in output (Y) in response to demand shocks (Δx); said more price stable (i.e., predictable Y) countries confuse relative demand shocks with aggregate demand shocks so Y (and u) are more likely to be adjusted the P

<u>Ball, Mankiw & Romer</u> - "The New Keynesian Economics and the Output-Inflation Tradeoff"; alternative explanation for swings in prices vs. output using menu costs (costly for firms to change prices so firms respond slowly to shocks; they rather change output than prices so inflation looks more like a step function)

Old Exam Questions

1. Recently, the possibility that developed economies may find themselves in a liquidity trap seems to have increased. How does a liquidity trap alter the degree to which a recession can be ended by:
(a) monetary policy?
(b) fiscal policy?
(c) a 'laissez faire' recovery?



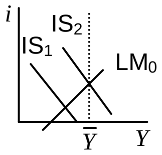


(a) $M\uparrow \Rightarrow$ LM$\uparrow$ (shift right); goal is to get $i\downarrow \Rightarrow I\uparrow \Rightarrow Y\uparrow$, but doesn't work because i can't be < 0 (graph on left)

(b) $G\uparrow \Rightarrow$ IS$\uparrow$ (shift right); this is not only effective (IS$_1$), but if government waits to act or uses monetary policy in conjunction (LM$_1$), the economy moves to potential output without impacting interest rates (IS$_2$; no crowding out) (graph on right)

(c) $P\downarrow \Rightarrow M/P\uparrow$ (i.e., LM$\uparrow$) $\Rightarrow i\downarrow \Rightarrow I\uparrow \Rightarrow Y\uparrow$; doesn't work for same reason as monetary policy (graph on left)

Which economy is in more danger of slipping into a liquidity trap? An economy with a
(d) high saving rate or low saving rate?
(e) high inflation rate or low inflation rate?



(d) high saving rate means less consumption so IS curve is lower (IS$_1$ vs. IS$_2$ for low saving rate) $\therefore$ <u>high</u> saving rate is in more danger

(e) high inflation rate means higher i so IS curve is higher (IS$_2$ vs. IS$_1$ for low inflation rate) $\therefore$ <u>low</u> inflation rate is in more danger

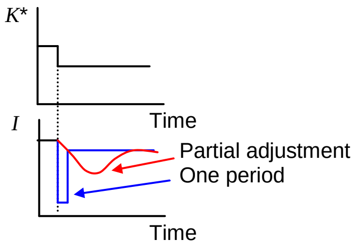
2. We have examined the work of three sets of authors on the effects of a sustained decrease in the inflation rate: Ball, Mankiw and Romer, Friedman, and Tobin. The models deal with the effect of lower inflation on: (1) the level of real money balances, (2) the position on the long-run Phillips Curve, (3) the slope of the short-run Phillips curve.
(a) which author deals with which issue? (b) n each case what is the effect of lower inflation and is it beneficial or not? (c) Explain the reasoning in each case.

Ball, Mankiw & Romer - did (3); argued that $\pi\downarrow \Rightarrow P$ adjusted less frequently so demand shocks goes into Y and u... bad

Friedman - did (1); argued that $\pi\downarrow \Rightarrow M/P\uparrow$ which is good because of the benefits of higher money balances (pecuniary services [shoe-leather costs] and non-pecuniary services [utility])

Tobin - did (2); argued that $\pi\downarrow \Rightarrow u\uparrow$ because short-run Phillips curve is curved from sticky wages; said low π is bad (higher π allows wages to respond quicker)

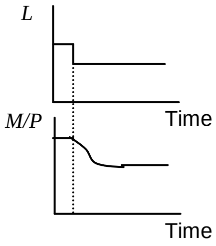
3. Suppose interest rate rise permanently.
(a) According to the "accelerator" specification of the investment function, will the initial decline in investment be larger than, smaller than, or of the same magnitude as the eventual decline in investment? Explain.



$I_t = K_t - K_{t-1} + \delta K_{t-1}$ (05 p.2)

If optimal level of capital doesn't change, investment = δK (i.e., just making up for depreciation); if the interest rates rise, the optimal level of capital will decline. In this case, investment will drop as firms let depreciation reduce the capital stock to the new optimal level. Eventually investment will pick up again to cover the depreciation of the new (lower) capital stock ∴ initial decline will be larger than eventual decline in I

(b) According to the "partial adjustment" specification commonly used in empirical work, will the initial decline in the demand for real money balances be larger than, smaller than or of the same magnitude as the eventual decline in the demand for real money balances? Explain.



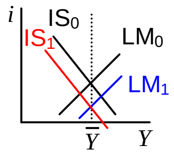
$\ln(M/P)_t = b_0 + b_1\ln(Y)_t + b_2\ln(i)_t + \ldots + c\ln(M/P)_{t-1}$ (06 p.7)

If L changes, there are costs for not being at L, but also for changing too quickly. People will adjust M/P slowly assuming there's some adjustment cost ∴ initial $\Delta M/P$ will be smaller than the total change in M/P

4. Currently the fraction of income American's save is low. Suppose this saving rate increases. Describe the short-run and long-run effects on:
(a) output, (b) investment, (c) interest rates, (d) exports

$S\uparrow \Rightarrow C\downarrow \Rightarrow$ IS↓ (shift left); short-run: $Y\downarrow$ & $i\downarrow \Rightarrow I\uparrow$; also looking at exchange rate from point of view as U.S. as home country, the \$ depreciates because interest rates are lower so capital flows move away from U.S.; dollar worth less so it takes more \$ to buy foreign currency: $E\uparrow$ (alternatively, fewer units of foreign currency to buy \$) $\Rightarrow EX\uparrow$ (**Note:** basically E and EX always move opposite the interest rate)

Long-run: $W\downarrow \Rightarrow$ LM↑ (shift right) $\Rightarrow Y\uparrow$ (net change is 0) and $i\downarrow$

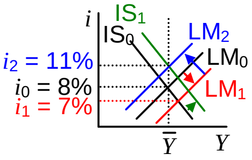


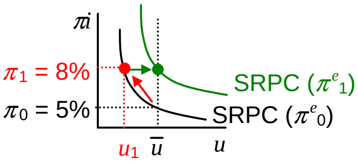
	S-R	L-R
Y	↓	--
i	↓	↓
I	↑	↑
E	↑	↑
EX	↑	↑

5. Use the expectations augmented Phillips Curve, developed by Milton Friedman and others, to describe the short-run and long-run effect of a sustained increase in the rate of money growth on (a) unemployment, (b) interest rates, (c) the inflation rate, (d) real interest rates

$\mu\uparrow \Rightarrow$ LM↑ (shifts right) $\Rightarrow$ short-run $u\downarrow$, $i\downarrow$, $\pi\uparrow$, $r\downarrow$... long-run $\pi^e\uparrow \Rightarrow$ IS↑ (shifts right), $u\uparrow$ (back to original); also $P\uparrow \Rightarrow$ LM↓ (shifts left) $\Rightarrow$ $i\uparrow$, $r\uparrow$ (back to original)

Long Version (with #'s): assume we start at i = 8%, u = 6% and $\pi = \pi^e$ = 5% ($\therefore$ $r = i - \pi^e$ = 8 - 5 = 3%), $\mu\uparrow$ by 3% $\Rightarrow$ $\pi\uparrow$ 3% (and π^e unchanged so $\pi > \pi^e$ so we move along Phillips Curve and $u\downarrow$); also from $\mu\uparrow$, LM curve shifts right lowering interest to say 7% (exact amount doesn't matter in short-run); note this means $r = i - \pi^e$ = 7 - 5 = 2%; $r\downarrow$); eventually $\pi^e\uparrow$ 3% to match π; this moves us to new short-run Phillips Curve and brings u back to original level; $\pi^e\uparrow$ also shifts IS curve to right which combined with LM shift left from $P\uparrow$ results in $i\uparrow$; final $\Delta i = \Delta\pi = \Delta\mu$ = 3%; $\therefore$ $r = i - \pi^e$ is unchanged





	S-R	L-R
u	↓	--
i	↓	↑
π	↑	↑
r	↓	--

6. (a) In his article on inflation and unemployment, James Tobin presented a model which implies that the central bank should aim an inflation rate which is greater than zero. Explain his reasoning.
(b) In his article "The Optimum Quantity of Money", Milton Friedman presented a model in which the optimal inflation rate is less than zero. Explain his reasoning.

For a & b see #2

(c) Suppose that Tobin is right that there is a benefit to having a positive inflation rate but Friedman is also right that there is a separate benefit to the proper amount of deflation. Explain which benefit is likely to be most important.

Lower u better than lower π... estimated benefit of 0 vs 5% π to be $60B... but by Okin's law 1%↓ in u results in 2%↑ in Y... that's more significant

7. According to estimates by Phillip Cagan, the spending which can be supported on a sustained basis by printing money reaches a maximum when money and price increase about 50% per year and the amount of spending which is thus financed is not very large as a portion of GDP.
(a) Why does so little spending lead to so much inflation?
(b) Why can't governments raise more revenue by increasing the money supply at an even faster rate?
(c) If this is the case, why do governments often produce inflation rates much higher than 50% per year?

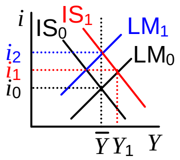
(a) Inflationary finance is effectively a tax on real money balances; in real terms government prints $\Delta M/P$... multiply by M/M and we get $(\Delta M/M)(M/P)$. That is $\mu = \Delta M/M$ (money growth rate) is the "tax rate" and M/P (real money balances) is tax base. End up with large inflation because large $\Delta\mu$ is needed since M/P (tax base) is small relative to GDP.

(b) As "tax rate" $(\Delta M/M)$ ↑, tax base (M/P) ↓... people try to avoid tax by holding less money.

(c) Trying to collect more money and can only do it if people don't expect $\pi\uparrow$

8. Recently, government purchases have risen in the U.S. Describe the short-run and long-run effects of this increase on: (a) output, (b) prices, (c) investment, (d) the exchange rate, (e) exports).

$G\uparrow \Rightarrow$ IS↑ (shift right); short-run: $Y\uparrow$ & $i\uparrow$; from AS-AD (graph not shown) $P\uparrow$; because of $i\uparrow$ I, E, and $EX\downarrow$ (see #4; Note: if asking about real exchange rate, $e\downarrow$ because $e = E/P$)
Long-run: $W\uparrow \Rightarrow$ LM↓ (shift left) $\Rightarrow Y\downarrow$ (net change is 0) and $i\uparrow$



	S-R	L-R
Y	↑	--
P	↑	↑
i	↑	↑
I	↓	↓
E	↓	↓
EX	↓	↓

How would your (short-run and long-run) answers change if the Federal Reserve were using monetary policy to keep prices from changing?

To counter $P\uparrow$, Fed would $M\downarrow \Rightarrow$ LM↓ (shift left)... basically automatically goes to long-run equilibrium (same as before except no change in P)
Long-run: already at long-run equilibrium

	S-R	L-R
Y	--	--
P	--	--
i	↑	↑
I	↓	↓
E	↓	↓
EX	↓	↓

ARIMA

2. Suppose that the rate of output (y_t) is defined as the change in the log of output (Y_t). That is, $y_t = Y_t - Y_{t\text{-}1}$. Suppose further than growth is described by the process:

$y_t = 3 + e_t$ where $e_t = u_t + 0.5u_{t\text{-}1}$ and u is a "white noise" error.

(a) if Yt is described as an ARIMA(P,D,Q), what are the values for P,D, and Q?
(b) If $Y_{2001} = 1000$ and $u_{2001} = -2$, what is the optimal forecast for Y_{2002}? Y_{2003}? Y_{2020}?

1 difference term (Y_t and $Y_{t\text{-}1}$) ∴ D = 1
1 moving average term ($u_{t\text{-}1}$) ∴ Q = 1
0 autoregressive terms (no lagged e) ∴ P = 0
Combine equations:
$Y_t = Y_{t\text{-}1} + 3 + u_t + 0.5u_{t\text{-}1}$
Remember that $E(u_t) = 0$ at time t because it's "white noise"
$Y_{2002} = Y_{2001} + 3 + E(u_{2002}) + 0.5u_{2001} = 1000 + 3 + 0 + 0.5(-2) =$ **1002**
Any period after 2002 will have $E(u_t)$ and $E(u_{t\text{-}1}) = 0$ (those periods haven't occurred)
$Y_{2003} = Y_{2002} + 3 + E(u_{2003}) + 0.5u_{2002} = 1002 + 3 + 0 + 0 =$ **1005**
So every period, Y will increase by 3 2020 is 17 years from 2003 ∴ $Y_{2020} = 1005 + 17(3) =$ **1056**

3. Suppose that output grows, on average at 3% per year with deviations above and below that trend:

$Y_t = Y_{t-1} + 0.03 + e_t$ (Y is the log of output).

Suppose further that deviations from trend fit the following pattern:
$e_t = u_t + 0.4e_{t-1}$ where u is a "white noise" error term.

Finally, suppose that recent values of Y are $Y_{2000} = 1$, $Y_{2001} = 1.05$, $Y_{2002} = 1.06$

(a) In the ARIMA classification this is an ARIMA(P,D,Q). What are the values for P, D, and Q? What is optimal conditional forecast for next year's Y: $E(Y_{2003})_{2002}$? $E(Y_{2004})_{2002}$? $E(Y_{2005})_{2002}$?

1 difference term (Y_t and Y_{t-1}) $\therefore$ D = 1
0 moving average term (No lagged u_t) $\therefore$ Q = 0
1 autoregressive terms (e_{t-1}) $\therefore$ P = 1
Use data we have to find error terms
$Y_{2001} = Y_{2000} + 0.03 + e_{2001} \Rightarrow e_{2001} = 1.05 - 1 - 0.03 = 0.02$
$Y_{2002} = Y_{2001} + 0.03 + e_{2002} \Rightarrow e_{2002} = 1.06 - 1.05 - 0.03 = -0.02$
Remember that $E(u_t) = 0$ at time t because it's "white noise"
$E(e_{2003})_{2002} = E(u_{2003})_{2002} + 0.4e_{2002} = 0 + 0.4(-0.02) = -0.008$
$E(Y_{2003})_{2002} = Y_{2002} + 0.03 + E(e_{2003})_{2002} = 1.06 + 0.03 -0.008 =$ **1.082**
Find next error term, then find Y_t
$E(e_{2004})_{2002} = E(u_{2004})_{2002} + 0.4e_{2003} = 0 + 0.4(-0.008) = -0.0032$
$E(Y_{2004})_{2002} = Y_{2003} + 0.03 + E(e_{2004})_{2002} = 1.082 + 0.03 -0.0032 =$ **1.1088**
$E(e_{2005})_{2002} = E(u_{2005})_{2002} + 0.4e_{2004} = 0 + 0.4(-0.0032) = -0.00128$
$E(Y_{2005})_{2002} = Y_{2004} + 0.03 + E(e_{2005})_{2002} = 1.1088 + 0.03 -0.00128 =$ **1.13752**

(b) Suppose instead $Y_t = Y_{t-1} + 0.03 + u_t$ where u is a "white noise" error term. How would your answers change?

1 difference term (Y_t and Y_{t-1}) $\therefore$ D = 1
0 moving average term (No lagged u_t) $\therefore$ Q = 0
0 autoregressive terms (No lagged e_t) $\therefore$ P = 0
Remember that $E(u_t) = 0$ at time t because it's "white noise"
$E(Y_{2003})_{2002} = Y_{2002} + 0.03 + E(u_{2003})_{2002} = 1.06 + 0.03 + 0 =$ **1.09**
$E(Y_{2004})_{2002} = Y_{2003} + 0.03 + E(u_{2004})_{2002} = 1.09 + 0.03 + 0 =$ **1.12**
$E(Y_{2005})_{2002} = Y_{2004} + 0.03 + E(u_{2005})_{2002} = 1.12 + 0.03 + 0 =$ **1.15**

1. Romer 5.1 ("Consider the IS-LM model presented in Section 5.1. In this model, what are di/dM and dY/dM for a given value of P?") Use the IS-LM version presented in class. Calculate short-run multipliers instead of fixed price multipliers.

Multiplier	Short-Run
$\frac{di}{dM}$	$\frac{(1-C')(F_N)^2}{Pz} < 0$
$\frac{dY}{dM}$	$\frac{(F_N)^2 I'}{Pz} > 0$

$$z = (F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P < 0$$

(work shown below)

4 Unknowns: Y, i, P, N
4 Equations:

Labor Market $\begin{cases} F_N = W/P \\ Y = F(K,N) \end{cases}$
Goods Market $Y = C(Y - T) + I(i - \pi_e) + G$
Asset Market $L(Y, i) = M/P$

Take total differentials

$$F_{NN}\,dN + F_{NK}\,dK = dW/P - (W/P^2)dP \tag{1}$$

$$dY = F_K\,dK + F_N\,dN \tag{2}$$

$$dY = C'dY - C'dT + I'di - I'd\pi_e + dG \tag{3}$$

$$L_Y\,dY + L_i\,di = dM/P - (M/P^2)dP \tag{4}$$

Solve Eqn (1) for dN

$$dN = \frac{-F_{NK}}{F_{NN}}dK + \frac{1}{PF_{NN}}dW + \frac{-W}{P^2F_{NN}}dP \tag{5}$$

Substitute this into Eqn (2)

$$dY = F_K\,dK + F_N\left[\frac{-F_{NK}}{F_{NN}}dK + \frac{1}{PF_{NN}}dW + \frac{-W}{P^2F_{NN}}dP\right]$$

$$= \frac{F_{NN}F_K - F_{NK}F_N}{F_{NN}}dK + \frac{F_N}{PF_{NN}}dW + \frac{-WF_N}{P^2F_{NN}}dP \tag{6}$$

Solve Eqn (3) for di

$$di = \frac{1-C'}{I'}dY + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG$$

Substitute dY from Eqn (6) into this one

$$di = \frac{1-C'}{I'}\left[\frac{F_{NN}F_K - F_{NK}F_N}{F_{NN}}dK + \frac{F_N}{PF_{NN}}dW + \frac{-WF_N}{P^2F_{NN}}dP\right] + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG$$

$$di = \frac{(1-C')(F_{NN}F_K - F_{NK}F_N)}{I'F_{NN}}dK + \frac{(1-C')F_N}{I'PF_{NN}}dW + \frac{-(1-C')WF_N}{I'P^2F_{NN}}dP + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG \tag{7}$$

Substitute dY from Eqn (6) and di from Eqn (7) into Eqn (4)

$$L_Y\left[\frac{F_{NN}F_K - F_{NK}F_N}{F_{NN}}dK + \frac{F_N}{PF_{NN}}dW + \frac{-WF_N}{P^2F_{NN}}dP\right] + L_i\left[\frac{(1-C')(F_{NN}F_K - F_{NK}F_N)}{I'F_{NN}}dK + \frac{(1-C')F_N}{I'PF_{NN}}dW + \frac{-(1-C')WF_N}{I'P^2F_{NN}}dP + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG\right] = dM/P - (M/P^2)\ dP$$

Multiply out L_Y and L_i

$$\frac{L_Y(F_{NN}F_K - F_{NK}F_N)}{F_{NN}}dK + \frac{L_YF_N}{PF_{NN}}dW + \frac{-L_YWF_N}{P^2F_{NN}}dP + \frac{L_i(1-C')(F_{NN}F_K - F_{NK}F_N)}{I'F_{NN}}dK + \frac{L_i(1-C')F_N}{I'PF_{NN}}dW + \frac{-L_i(1-C')WF_N}{I'P^2F_{NN}}dP + \frac{L_iC'}{I'}dT + L_id\pi_e + \frac{-L_i}{I'}dG = dM/P - (M/P^2)\ dP$$

Combine all dP terms on the left side and all others terms on the right

$$\left[\frac{L_YWF_N}{P^2F_{NN}} + \frac{L_i(1-C')WF_N}{I'P^2F_{NN}} + \frac{-M}{P^2}\right]dP = \frac{L_iC'}{I'}dT + L_id\pi_e + \frac{-L_i}{I'}dG + \frac{-1}{P}dM + \left[\frac{L_Y(F_{NN}F_K - F_{NK}F_N)}{F_{NN}} + \frac{L_i(1-C')(F_{NN}F_K - F_{NK}F_N)}{I'F_{NN}}\right]dK + \left[\frac{L_YF_N}{PF_{NN}} + \frac{L_i(1-C')F_N}{I'PF_{NN}}\right]dW \tag{8}$$

Just focus on the left side of Eqn (8) and substitute $W = PF_N$

$$\left[\frac{L_Y(PF_N)F_N}{P^2F_{NN}} + \frac{L_i(1-C')(PF_N)F_N}{I'P^2F_{NN}} + \frac{-M}{P^2}\right]dP$$

Get a common denominator ($I'P^2F_{NN}$)

$$\left[\frac{I'L_Y(PF_N)F_N}{I'P^2F_{NN}} + \frac{L_i(1-C')(PF_N)F_N}{I'P^2F_{NN}} + \frac{-I'F_{NN}M}{I'P^2F_{NN}}\right]dP$$

Multiply the F_N terms and cancel the P in the first two terms with P^2 in the denominator; in the third term, we get a P (rather than P^2) by moving one of the Ps up to the numerator as $1/P$

$$\left[\frac{I'L_Y(F_N)^2}{I'PF_{NN}} + \frac{L_i(1-C')(F_N)^2}{I'PF_{NN}} + \frac{-I'F_{NN}M/P}{I'PF_{NN}}\right]dP$$

Now add the terms together and factor the $(F_N)^2$

$$\left[\frac{(F_N)^2(L_i(1-C') + I'L_Y) - I'F_{NN}M/P}{I'PF_{NN}}\right]dP$$

Define a new variable z as the numerator and rewrite the expression

$$\left[\frac{z}{I'PF_{NN}}\right]dP \text{ , where} \tag{9}$$

$$\boxed{z = (F_N)^2(L_i(1-C')+I'L_Y)-F_{NN}I'M/P < 0} \tag{10}$$

Signs: $(+)^2$ [(- +) + (-+)] - - - + / + = (+-) - + = - - +; a negative value minus a positive value will always be negative

Before looking at the entire right side of Eqn (8), just consider the dK term

$$\left[\frac{L_Y(F_{NN}F_K-F_{NK}F_N)}{F_{NN}}+\frac{L_i(1-C')(F_{NN}F_K-F_{NK}F_N)}{I'F_{NN}}\right]dK$$

Get a common denominator by multiplying the first term by I'

$$\left[\frac{I'L_Y(F_{NN}F_K-F_{NK}F_N)}{I'F_{NN}}+\frac{L_i(1-C')(F_{NN}F_K-F_{NK}F_N)}{I'F_{NN}}\right]dK$$

Add the terms together and factor the ($F_{NK}F_K$ - $F_N K_{FN}$) term

$$\left[\frac{(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{I'F_{NN}}\right]dK \tag{11}$$

Now look at the dW term from the right side of Eqn (8)

$$\left[\frac{L_YF_N}{PF_{NN}}+\frac{L_i(1-C')F_N}{I'PF_{NN}}\right]dW$$

Get a common denominator by multiplying the first term by I'

$$\left[\frac{I'L_YF_N}{I'PF_{NN}}+\frac{L_i(1-C')F_N}{I'PF_{NN}}\right]dW$$

Add the terms together and factor the F_N term

$$\left[\frac{F_N(L_i(1-C')+I'L_Y)}{I'PF_{NN}}\right]dW \tag{12}$$

Now simplify the right side of Eqn (8) by substituting Eqns (11) and (12)

$$\frac{L_iC'}{I'}dT + L_i d\pi_e + \frac{-L_i}{I'}dG + \frac{-1}{P}dM + \frac{(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{I'F_{NN}}dK + \frac{F_N(L_i(1-C')+I'L_Y)}{I'PF_{NN}}dW \tag{13}$$

Use the results of Eqns (9) and (13) to solve for dP

$$dP = \frac{I'PF_{NN}}{z}\left[\frac{L_iC'}{I'}dT+L_id\pi_e+\frac{-L_i}{I'}dG+\frac{-1}{P}dM + \frac{F_N(L_i(1-C')+I'L_Y)}{I'PF_{NN}}dW + \frac{(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{I'F_{NN}}dK\right]$$

$$dP = \boxed{\frac{PF_{NN}L_iC'}{z}}\,dT + \boxed{\frac{I'PF_{NN}L_i}{z}}\,d\pi_e + \boxed{\frac{-PF_{NN}L_i}{z}}\,dG + \boxed{\frac{-I'F_{NN}}{z}}\,dM + \boxed{\frac{F_N(L_i(1-C')+I'L_Y)}{z}}\,dW + \boxed{\frac{P(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{z}}\,dK \tag{14}$$

Substitute this value of dP into Eqn (7) to solve for di

$$di = \frac{(1-C')(F_{NN}F_K-F_{NK}F_N)}{I'F_{NN}}dK + \frac{(1-C')F_N}{I'PF_{NN}}dW + \frac{-(1-C')WF_N}{I'P^2F_{NN}}\left[\frac{PF_{NN}L_iC'}{z}\,dT + \frac{I'PF_{NN}L_i}{z}\,d\pi_e + \frac{-PF_{NN}L_i}{z}\,dG + \frac{-I'F_{NN}}{z}\,dM + \frac{F_N(L_i(1-C')+I'L_Y)}{z}\,dW + \frac{P(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{z}dK\right] + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG \tag{15}$$

To make this easier, look at only the dK terms first and substitute $W = PF_N$

$$\left[\frac{(1-C')(F_{NN}F_K-F_{NK}F_N)}{I'F_{NN}} + \frac{-(1-C')(PF_N)F_NP(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{I'P^2F_{NN}z}\right]dK$$

In the second term, the P^2 cancels so all we have to do is multiply the first term by 1 = z/z to get a common denominator

$$\left[\frac{(1-C')(F_{NN}F_K-F_{NK}F_N)z-(1-C')(F_N)^2(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{I'F_{NN}z}\right]dK$$

The numerator can be further simplified by substituting z from Eqn (10) in the numerator

$$\left[\frac{(1-C')(F_{NN}F_K-F_{NK}F_N)\left[(F_N)^2(L_i(1-C')+I'L_Y)-F_{NN}I'M/P\right]}{I'F_{NN}z} + \frac{-(1-C')(F_N)^2(F_{NN}F_K-F_{NK}F_N)(L_i(1-C')+I'L_Y)}{I'F_{NN}z}\right]dK$$

Notice that when we multiply the first term of z by the expression outside the brackets, it cancels with the second term in the sum, so now we have

$$\left[\frac{-(1-C')(F_{NN}F_K-F_{NK}F_N)F_{NN}I'M/P}{I'F_{NN}z}\right]dK$$

The $I'F_{NN}$ terms cancel so the final expression for dK is

$$\left[\frac{-(1-C')(F_{NN}F_K-F_{NK}F_N)M}{Pz}\right]dK \tag{16}$$

Again to make Eqn (15) easier to work with, let's look only at the dW terms and substitute $W = PF_N$

$$\left[\frac{(1-C')F_N}{I'PF_{NN}} + \frac{-(1-C')(PF_N)F_N{}^2(L_i(1-C')+I'L_Y)}{I'P^2F_{NN}z}\right]dW$$

In the second term, the P cancels so all we have to do is multiply the first term by 1 = z/z to get a common denominator

$$\left[\frac{(1-C')F_N z-(1-C')F_N{}^3(L_i(1-C')+I'L_Y)}{I'PF_{NN}z}\right]dW$$

The numerator can be further simplified by substituting z from Eqn (10) in the numerator

$$\left[\frac{(1-C')F_N\left[(F_N)^2(L_i(1-C')+I'L_Y)-F_{NN}I'M/P\right]-(1-C')F_N{}^3(L_i(1-C')+I'L_Y)}{I'PF_{NN}z}\right]dW$$

Notice that when we multiply the first term of z by the expression outside the brackets, it cancels with the second term in the sum, so now we have

$$\left[\frac{-(1-C')F_N F_{NN}I'M/P}{I'PF_{NN}z}\right]dW$$

No we can cancel the $I'F_{NN}$ terms and move the P to the denominator to get our final expression for dW

$$\left[\frac{-(1-C')F_N M}{P^2 z}\right]dW \tag{17}$$

We now substitute Eqns (16) and (17) into Eqn (15) and multiply out the other terms in the brackets

$$\begin{aligned}di = &\left[\frac{-(1-C')(F_{NN}F_K-F_{NK}F_N)M}{Pz}\right]dK+\left[\frac{-(1-C')F_N M}{P^2 z}\right]dW+\\ &\left[\frac{-(1-C')WF_N PF_{NN}L_iC'}{I'P^2F_{NN}z}+\frac{C'}{I'}\right]dT+\left[\frac{-(1-C')WF_N I'PF_{NN}L_i}{I'P^2F_{NN}z}+1\right]d\pi_e+\\ &\left[\frac{(1-C')WF_N PF_{NN}L_i}{I'P^2F_{NN}z}+\frac{-1}{I'}\right]dG+\left[\frac{(1-C')WF_N I'F_{NN}}{I'P^2F_{NN}z}\right]dM\end{aligned}$$

For the terms we have not already simplified (dT, $d\pi_e$, dG, and dM), we now perform similar tasks as we did with dK and dW. First, substitute $W = PF_N$,

$$\begin{aligned}di = &\left[\frac{-(1-C')(F_{NN}F_K-F_{NK}F_N)M}{Pz}\right]dK+\left[\frac{-(1-C')F_N M}{P^2 z}\right]dW+\\ &\left[\frac{-(1-C')(PF_N)F_N PF_{NN}L_iC'}{I'P^2F_{NN}z}+\frac{C'}{I'}\right]dT+\left[\frac{-(1-C')(PF_N)F_N I'PF_{NN}L_i}{I'P^2F_{NN}z}+1\right]d\pi_e+\\ &\left[\frac{(1-C')(PF_N)F_N PF_{NN}L_i}{I'P^2F_{NN}z}+\frac{-1}{I'}\right]dG+\left[\frac{(1-C')(PF_N)F_N I'F_{NN}}{I'P^2F_{NN}z}\right]dM\end{aligned}$$

Now simplify each expression

$$\begin{aligned}di = &\left[\frac{-(1-C')(F_{NN}F_K-F_{NK}F_N)M}{Pz}\right]dK+\left[\frac{-(1-C')F_N M}{P^2 z}\right]dW+\\ &\left[\frac{-(1-C')(F_N)^2L_iC'}{I'z}+\frac{C'}{I'}\right]dT\left[\frac{-(1-C')(F_N)^2L_iC'}{I'z}+\frac{C'}{I'}\right]dT+\\ &\left[\frac{-(1-C')(F_N)^2L_i}{z}+1\right]d\pi_e+\left[\frac{(1-C')(F_N)^2L_i}{I'z}+\frac{-1}{I'}\right]dG+\left[\frac{(1-C')(F_N)^2}{Pz}\right]dM\end{aligned}$$

The second step we did with dK and dW was to get common denominators. We do that now for the other four terms

$$di = \left[\frac{-(1-C')(F_{NN}F_K - F_{NK}F_N)M}{Pz}\right]dK + \left[\frac{-(1-C')F_N M}{P^2 z}\right]dW +$$

$$\left[\frac{-(1-C')(F_N)^2 L_i C'}{I'z} + \frac{C'z}{I'z}\right]dT + \left[\frac{-(1-C')(F_N)^2 L_i}{z} + \frac{z}{z}\right]d\pi_e +$$

$$\left[\frac{(1-C')(F_N)^2 L_i I'}{I'z} + \frac{-z}{I'z}\right]dG + \left[\frac{(1-C')(F_N)^2}{Pz}\right]dM \tag{18}$$

Now substitute z from Eqn (10) into the numerators of dT, $d\pi_e$, and dG and simplify the expressions (each done individually)

dT:

$$\left[\frac{-(1-C')(F_N)^2 L_i C' + C'\left[(F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P\right]}{I'z}\right]dT =$$

$$\left[\frac{-(1-C')(F_N)^2 L_i C' + C'(F_N)^2 L_i(1-C') + C'(F_N)^2 I'L_Y - C'F_{NN}I'M/P}{I'z}\right]dT =$$

$$\left[\frac{C'(F_N)^2 I'L_Y - C'F_{NN}I'M/P}{I'z}\right]dT =$$

$$\left[\frac{C'I'\left[(F_N)^2 L_Y - F_{NN}M/P\right]}{I'z}\right]dT =$$

$$\left[\frac{C'\left[(F_N)^2 L_Y - F_{NN}M/P\right]}{z}\right]dT \tag{19}$$

$d\pi_e$:

$$\left[\frac{-(1-C')(F_N)^2 L_i + \left[(F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P\right]}{z}\right]d\pi_e =$$

$$\left[\frac{-(1-C')(F_N)^2 L_i + (F_N)^2 L_i(1-C') + (F_N)^2 I'L_Y - F_{NN}I'M/P}{z}\right]d\pi_e =$$

$$\left[\frac{(F_N)^2 I'L_Y - F_{NN}I'M/P}{z}\right]d\pi_e =$$

$$\left[\frac{I'\left[(F_N)^2 L_Y - F_{NN}M/P\right]}{z}\right]d\pi_e \tag{20}$$

dG:

$$\left[\frac{(1-C')(F_N)^2 L_i I' - \left[(F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P\right]}{I'z}\right]dG =$$

$$\left[\frac{(1-C')(F_N)^2 L_i I' - (F_N)^2 L_i(1-C') - (F_N)^2 I'L_Y + F_{NN}I'M/P}{I'z}\right]dG =$$

$$\left[\frac{-(F_N)^2 I' L_Y + F_{NN} I' M / P}{I' z}\right] dG \ =$$
$$\left[\frac{I'\left[F_{NN} M / P - (F_N)^2 L_Y\right]}{I' z}\right] dG \ =$$
$$\left[\frac{F_{NN} M / P - (F_N)^2 L_Y}{z}\right] dG \tag{21}$$

Now substitute Eqns (19), (20), and (21) back into Eqn (18)

$$di = \boxed{\frac{-(1-C')(F_{NN} F_K - F_{NK} F_N) M}{Pz}}\ dK + \boxed{\frac{-(1-C') F_N M}{P^2 z}}\ dW +$$
$$\boxed{\frac{C'\left[(F_N)^2 L_Y - F_{NN} M / P\right]}{z}}\ dT + \boxed{\frac{I'\left[(F_N)^2 L_Y - F_{NN} M / P\right]}{z}}\ d\pi_e +$$
$$\boxed{\frac{F_{NN} M / P - (F_N)^2 L_Y}{z}}\ dG + \boxed{\frac{(1-C')(F_N)^2}{Pz}}\ dM \tag{22}$$

Substitute dP from Eqn (14) to solve for dY in Eqn (6)

$$dY = \frac{F_{NN} F_K - F_{NK} F_N}{F_{NN}} dK + \frac{F_N}{PF_{NN}} dW + \frac{-WF_N}{P^2 F_{NN}} \frac{PF_{NN} L_i C'}{z} dT \ +$$
$$\frac{-WF_N}{P^2 F_{NN}} \frac{I' PF_{NN} L_i}{z} d\pi_e \ + \frac{-WF_N}{P^2 F_{NN}} \frac{-PF_{NN} L_i}{z} dG + \frac{-WF_N}{P^2 F_{NN}} \frac{-I' F_{NN}}{z} dM +$$
$$\frac{-WF_N}{P^2 F_{NN}} \frac{F_N (L_i (1-C') + I' L_Y)}{z} dW +$$
$$\frac{-WF_N}{P^2 F_{NN}} \frac{P(F_{NN} F_K - F_{NK} F_N)(L_i (1-C') + I' L_Y)}{z} dK$$

Combine terms and substitute $W = PF_N$

$$dY = \frac{(F_{NN} F_K - F_{NK} F_N) z - (F_N)^2 (F_{NN} F_K - F_{NK} F_N)(L_i (1-C') + I' L_Y)}{F_{NN} z} dK +$$
$$\frac{F_N z - (F_N)^3 (L_i (1-C') + I' L_Y)}{PF_{NN} z} dW + \frac{-(F_N)^2 L_i C'}{z} dT \ + \frac{-(F_N)^2 I' L_i}{z} d\pi_e \ +$$
$$\frac{(F_N)^2 L_i}{z} dG + \frac{(F_N)^2 I'}{Pz} dM$$

Now plug in z from Eqn (9) in the numerators and do some more Algebra to get

$$dY = \boxed{\frac{(F_{NN} F_K - F_{NK} F_N) I' M}{Pz}}\ dK + \boxed{\frac{-F_N I' M}{P^2 z}}\ dW + \boxed{\frac{-(F_N)^2 L_i C'}{z}}\ dT \ +$$
$$\boxed{\frac{-(F_N)^2 I' L_i}{z}}\ d\pi_e \ + \boxed{\frac{(F_N)^2 L_i}{z}}\ dG + \boxed{\frac{(F_N)^2 I'}{Pz}}\ dM \tag{23}$$

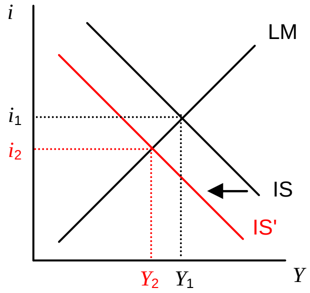
Substitute dP from Eqn (14) to solve for dN in Eqn (5)... but don't need to do that for this assignment!

2. Romer 5.5 ("**The Mundell effect.** (Mundell, 1963.) in the IS-LM model, how does a fall in expected inflation π_e affect i, Y, and i - π_e?") Use the IS-LM version presented in class. Calculate short-run multipliers instead of fixed price multipliers.

Multiplier	Short-Run	$\pi_e \downarrow \Rightarrow$
$\frac{di}{d\pi_e}$	$\frac{I'(L_Y(F_N)^2 - F_{NN}M/P)}{z} > 0$	$\downarrow$
$\frac{dY}{d\pi_e}$	$\frac{-(F_N)^2 I' L_i}{z} > 0$	$\downarrow$
$\frac{d(i-\pi_e)}{d\pi_e}$	$\frac{-(F_N)^2 L_i(1-C')}{z} < 0$	$\uparrow$

$$z = (F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P < 0$$

(work shown below)



If people expect inflation to fall, **interest rates (*i*) and output (*Y*) will also fall** according to the multipliers calculated in problem 1. This makes sense since a fall in expected inflation causes leftward shift in the IS curve (see graph). An intuitive explanation for this effect comes from business investment. If firms believe inflation will fall, but has not done so already, current nominal interest rates (i) for loans will be higher than the firms think they should be. This means they will put off investment purchases until nominal rates come in line with their expectations. While firms put off investments, output ($Y = C + I + G$) will fall. Interest rates will also fall as a result of fewer firms taking loans to finance their investments.

$$\frac{dr}{d\pi_e} = \frac{d(i-\pi_e)}{d\pi_e} = \frac{di}{d\pi_e} - \frac{d\pi_e}{d\pi_e} = \frac{di}{d\pi_e} - 1$$

Substitute $di/d\pi_e$ from above

$$\frac{d(i-\pi_e)}{d\pi_e} = \frac{I'(L_Y(F_N)^2 - F_{NN}M/P)}{z} - 1$$

Use 1 = z/z and substitute z from Eqn (9) into the numerator

$$\frac{d(i-\pi_e)}{d\pi_e} = \frac{I'(L_Y(F_N)^2 - F_{NN}M/P)}{z} - \frac{(F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P}{z}$$

Simplify

$$\frac{d(i-\pi_e)}{d\pi_e} = \frac{I'L_Y(F_N)^2 - I'F_{NN}M/P - (F_N)^2 L_i(1-C') - (F_N)^2 I'L_Y + F_{NN}I'M/P}{z}$$

$$\boxed{\frac{d(i-\pi_e)}{d\pi_e} = \frac{-(F_N)^2 L_i(1-C')}{z} < 0}$$

This result means that real interest rates move in the opposite direction as expected inflation. Specifically for this case, if people expect inflation to fall, **real interest rates ($r = i - \pi_e$) will increase**. This result also follows directly from $di/d\pi_e$ because a little algebra verifies that $|di/d\pi_e| < 1$. That is, if expected inflation (π_e) falls, interest rates (i) will fall, but by less than expected inflation. Therefore, $r = i - \pi_e$ will actually increase.

3. Instead of regarding M as exogenous and P as endogenous, treat P as exogenous and M as endogenous. Calculate short-run and long-run multipliers dY/dG, di/dG, and dM/dG for the new system and determine their signs.

Multiplier	Short-Run	Long-Run
$\frac{dY}{dG}$	0	0
$\frac{di}{dG}$	$\frac{-1}{I'} > 0$	$\frac{-1}{I'} > 0$
$\frac{dM}{dG}$	$\frac{-PL_i}{I'} < 0$	$\frac{-PL_i}{I'} < 0$

(work shown below)

Long-Run

5 Unknowns: Y, i, M, N, W (can't use w because W is endogenous & P is exogenous)

5 Equations:

Labor Market $\left\{\begin{array}{l} F_N = W/P \\ N = \bar{N} \\ Y = F(K,N) \end{array}\right.$

Goods Market $Y = C(Y - T) + I(i - \pi_e) + G$

Asset Market $L(Y, i) = M/P$

Take total differentials

$$F_{NN}\, dN + F_{NK}\, dK = dW/P - (W/P^2)dP \tag{1}$$

$$dN = d\bar{N} \tag{2}$$

$$dY = F_K\, dK + F_N\, dN \tag{3}$$

$$dY = C'dY - C'dT + I'di - I'd\pi_e + dG \tag{4}$$

$$L_Y\, dY + L_i\, di = dM/P - (M/P^2)dP \tag{5}$$

Eqn (2) is already solved

Substitute dN into Eqn (3) to solve for dY

$$dY = F_K\, dK + F_N\, d\bar{N} \quad (\textbf{Note:}\ \boxed{dY/dG = 0}) \tag{6}$$

Solve Eqn (4) for di

$$di = \frac{1-C'}{I'}dY + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG$$

Substitute dY from Eqn (6) into this one

$$di = \frac{1-C'}{I'}\left[F_K\, dK + F_N\, d\bar{N}\right] + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG$$

$$di = \frac{(1-C')F_K}{I'}dK + \frac{(1-C')F_N}{I'}d\overline{N} + \frac{C'}{I'}dT + d\pi_e + \boxed{\frac{-1}{I'}}dG \tag{7}$$

Substitute dY from Eqn (6) and di from Eqn (7) into Eqn (4)

$$L_Y\left[F_K dK + F_N d\overline{N}\right] + L_i\left[\frac{(1-C')F_K}{I'}dK + \frac{(1-C')F_N}{I'}d\overline{N} + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG\right] =$$
$$dM/P - (M/P^2)\, dP$$

Solve for dM. We're only interested in the dG term so ignore the others.

$$dM = [\ ]dK + [\ ]\,d\overline{N} + [\ ]dT + [\ ]d\pi_e + [\ ]dP + \boxed{\frac{-PL_i}{I'}}dG \tag{8}$$

Short-Run

4 Unknowns: Y, i, M, N

4 Equations:

Labor Market $\begin{cases} F_N = W/P = w \text{ (can use } w \text{ since } W \text{ \& } P \text{ are exogenous)} \\ Y = F(K,N) \end{cases}$

Goods Market $Y = C(Y - T) + I(i - \pi_e) + G$

Asset Market $L(Y, i) = M/P$

Take total differentials

$$F_{NN}\,dN + F_{NK}\,dK = dw \tag{1}$$
$$dY = F_K\,dK + F_N\,dN \tag{2}$$
$$dY = C'dY - C'dT + I'di - I'd\pi_e + dG \tag{3}$$
$$L_Y\,dY + L_i\,di = dM/P - (M/P^2)dP \tag{4}$$

Solve Eqn (1) for dN

$$dN = \frac{-F_{NK}}{F_{NN}}dK + \frac{1}{F_{NN}}dw \tag{5}$$

Substitute this into Eqn (2)

$$dY = F_K\,dK + F_N\left[\frac{-F_{NK}}{F_{NN}}dK + \frac{1}{F_{NN}}dw\right] = \frac{F_{NN}F_K - F_{NK}F_N}{F_{NN}}dK + \frac{F_N}{F_{NN}}w \tag{6}$$

(**Note:** $\boxed{dY/dG = 0}$)

Solve Eqn (3) for di

$$di = \frac{1-C'}{I'}dY + \frac{C'}{I'}dT + d\pi_e + \frac{-1}{I'}dG$$

We could substitute dY from Eqn (6) into this one, but there's no dG term so it's not important

$$di = [\ ]dK + [\ ]dw + \frac{C'}{I'}dT + d\pi_e + \boxed{\frac{-1}{I'}}dG \tag{7}$$

Substitute dY from Eqn (6) and di from Eqn (7) into Eqn (4)

$$[\ ]dK + [\ ]dw + [\ ]dT + [\ ]d\pi_e + \frac{-L_i}{I'}dG = dM/P - (M/P^2)\, dP$$

Solve for dM. We're only interested in the dG term so ignore the others.

$$dM = [\ ]dK + [\ ]\,d\overline{N} + [\ ]dT + [\ ]d\pi_e + [\ ]dP + \boxed{\frac{-PL_i}{I'}}dG \tag{8}$$

4. Instead of regarding M as exogenous and i as endogenous, treat M as endogenous and i as exogenous. Calculate short-run multipliers dY/dG, dI/dG, dM/dG, $dY/d\pi_e$, $dI/d\pi_e$, and $dM/d\pi_e$ and determine their signs.

Multiplier	Short-Run
$\frac{dY}{dG}$	$\frac{1}{1-C'} > 0$
$\frac{dI}{dG}$	0
$\frac{dM}{dG}$	$\frac{P(WF_N L_Y - MF_{NN})}{WF_N(1-C')} > 0$
$\frac{dY}{d\pi_e}$	$\frac{-I'}{1-C'} > 0$
$\frac{dI}{d\pi_e}$	$-I' > 0$
$\frac{dM}{d\pi_e}$	$\frac{I'P(MF_{NN} - WF_N L_Y)}{WF_N(1-C')} > 0$

(work shown below)

4 Unknowns: Y, M, P, N

4 Equations:

Labor Market $\begin{cases} F_N = W/P \\ Y = F(K,N) \end{cases}$

Goods Market $Y = C(Y - T) + I(i - \pi_e) + G$

Asset Market $L(Y, i) = M/P$

Take total differentials

$$F_{NN}\,dN + F_{NK}\,dK = dW/P - (W/P^2)dP \qquad (1)$$

$$dY = F_K\,dK + F_N\,dN \qquad (2)$$

$$dY = C'dY - C'dT + I'di - I'd\pi_e + dG \qquad (3)$$

$$L_Y\,dY + L_i\,di = dM/P - (M/P^2)dP \qquad (4)$$

Throughout this problem, we'll ignore all exogenous terms other than dG and $d\pi_e$.

Solve Eqn (3) for dY

$$dY = [\ ]dT + [\ ]di + \boxed{\frac{-I'}{1-C'}}\,d\pi_e + \boxed{\frac{1}{1-C'}}\,dG \qquad (5)$$

Substitute this into Eqn (2) and solve for dN

$$dN = [\ ]dK + \frac{1}{F_N}dY = [\ ]dK + [\ ]dT + [\ ]di + \frac{-I'}{F_N(1-C')}\,d\pi_e + \frac{1}{F_N(1-C')}\,dG \qquad (6)$$

Solve Eqn (1) for dP

$$dP = [\ ]dW + [\ ]dK + \frac{-P^2F_{NN}}{W}\,dN$$

Substitute dN from Eqn (6) into this equation for dP

$$dP = [\,]dW + [\,]dK + [\,]dT + [\,]di + \frac{P^2F_{NN}I'}{WF_N(1-C')}d\pi_e + \frac{-P^2F_{NN}}{WF_N(1-C')}dG \tag{7}$$

Solve Eqn (4) for dM

$$dM = [\,]di + PL_YdY + \frac{M}{P}dP$$

Substitute dY from Eqn (5) and dP from Eqn (7) into this equation for dM

$$dM = [\,]di + [\,]dT + \frac{-I'PL_Y}{1-C'}d\pi_e + \frac{PL_Y}{1-C'}dG + [\,]dW + [\,]dK + \frac{MPF_{NN}I'}{WF_N(1-C')}d\pi_e + \frac{-MPF_{NN}}{WF_N(1-C')}dG$$

Combine the dG and $d\pi_e$ terms

$$dM = [\,]di + [\,]dT + [\,]dW + [\,]dK + \boxed{\frac{I'P(MF_{NN} - WF_NL_Y)}{WF_N(1-C')}}d\pi_e + \boxed{\frac{P(WF_NL_Y - MF_{NN})}{WF_N(1-C')}}dG \tag{8}$$

Now to get the dI terms realized that

$$dI = I'di - I'd\pi_e \tag{9}$$

Since there is no dG term in Eqn (9), $\boxed{dI/dG = 0}$

To find $dI/d\pi_e$, divide Eqn (9) by $d\pi_e$

$$\frac{dI}{d\pi_e} = I'\frac{di}{d\pi_e} - I'\frac{d\pi_e}{d\pi_e} = I'(0) - I'(1) = \boxed{-I'}$$

5. Compare dY/dG and di/dG for (a) fixed prices, (b) the short-run, and (c) the long-run. Which version is largest, smallest, and in between?

(a) $\frac{dY}{dG} = \frac{L_i}{L_i(1-C') + I'L_Y} > 0$ $\qquad \frac{di}{dG} = \frac{-L_Y}{L_i(1-C') + I'L_Y} > 0$

(b) $\frac{dY}{dG} = \frac{(F_N)^2L_i}{z} > 0$ $\qquad \frac{di}{dG} = \frac{F_{NN}M/P - L_Y(F_N)^2}{z} > 0$

(c) $\frac{dY}{dG} = 0$ $\qquad \frac{di}{dG} = \frac{-1}{I'} > 0$

dY/dG

Obviously, (a) and (b) are greater than (c) since (a) > 0, (b) > 0, and (c) = 0

To check the relative magnitudes of (a) and (b), substitute z from Eqn (9) into (b)

$$\frac{dY}{dG}\text{(b)} = \frac{(F_N)^2L_i}{z} = \frac{(F_N)^2L_i}{(F_N)^2(L_i(1-C') + I'L_Y) - F_{NN}I'M/P}$$

If we ignore the second term in the denominator ($F_{NN}I'M/P$), the $(F_N)^2$ terms cancel and (b) is the same as (a). The first term in the denominator

$[(F_N)^2(L_i(1 - C') + I'L_Y)]$ is negative and the second term $[F_{NN}I'M/P]$ is positive. Subtracting a positive from a negative, increases the absolute value of the denominator. Therefore, dividing by a larger number in (b) results in a smaller multiplier relative to (a).

Summary: **(a) > (b) > (c)** (i.e., fixed-price > short-run > long-run)

di/dG

To check the relative magnitudes of (b) and (c), multiply the numerator of (b) by -1 and rearrange the terms in the denominator to get:

$$\frac{di}{dG}\text{(b)} = \frac{-\left(L_Y(F_N)^2 - F_{NN}M/P\right)}{(F_N)^2 L_i(1-C') + I'\left(L_Y(F_N)^2 - F_{NN}M/P\right)}$$

If we ignore the $(F_N)^2L_i(1 - C')$ term in the denominator, this equation is the same as equation (c). By adding the term back in, we are increasing the absolute value of the denominator (since both terms are negative). This means (b) < (c).

Looking at equation (a), we can use the same argument to conclude (a) < (c). In order to determine the relative magnitudes of (a) and (b), we need to consider the difference between adding $(F_N)^2L_i(1 - C')$ to equation (b) and $L_i(1 - C')$ to equation (a). This can't be determined mathematically without knowing more about F_N. Based on class notes, however, the long-run impact on interest rates will be greater than the short-run impact so (c) > (b).

Summary: **(c) > (b) > (a)** (i.e., long-run > short-run > fixed-price)

Documentation

Prof Bomberger helped immensely on this assignment. For problem 1, he pointed out that short-run multipliers are the same as the fixed-wage multipliers we did in class. He also suggest the order for solving the equations to isolate the four unknowns. For problem 2, Prof Bomberger went over the logic behind the signs of the multipliers. He also cleared up $d(i - \pi_e)/d\pi_e = di/d\pi_e - 1$. Prof Bomberger gave me general advice on doing problems 3 and 4; basically, I should ignore any terms with exogenous variables the problems are not asking for. For problem 5, Prof Bomberger verified my intuition on the answers and reminded me that we did a similar problem in class to determine the magnitudes mathematically.

I checked my work with Guille Sabbioni. Specifically, Guille helped with dI/dG and $dI/d\pi_e$ in problem 4 by reminding me that $dI = I'di - I'd\pi_e$.

Be sure to illustrate your answers with (a) IS-LM curves, (b) AD-AS curves, and (c) labor demand and labor supply curves

Notes on notation:

A. *↓ means * is decreasing or shifting to the left; *↑ means * is increasing or shifting to the right; *? means * cannot be determined; -- means no change

B. Black is the starting condition; red represents short-run; green represent the long-run

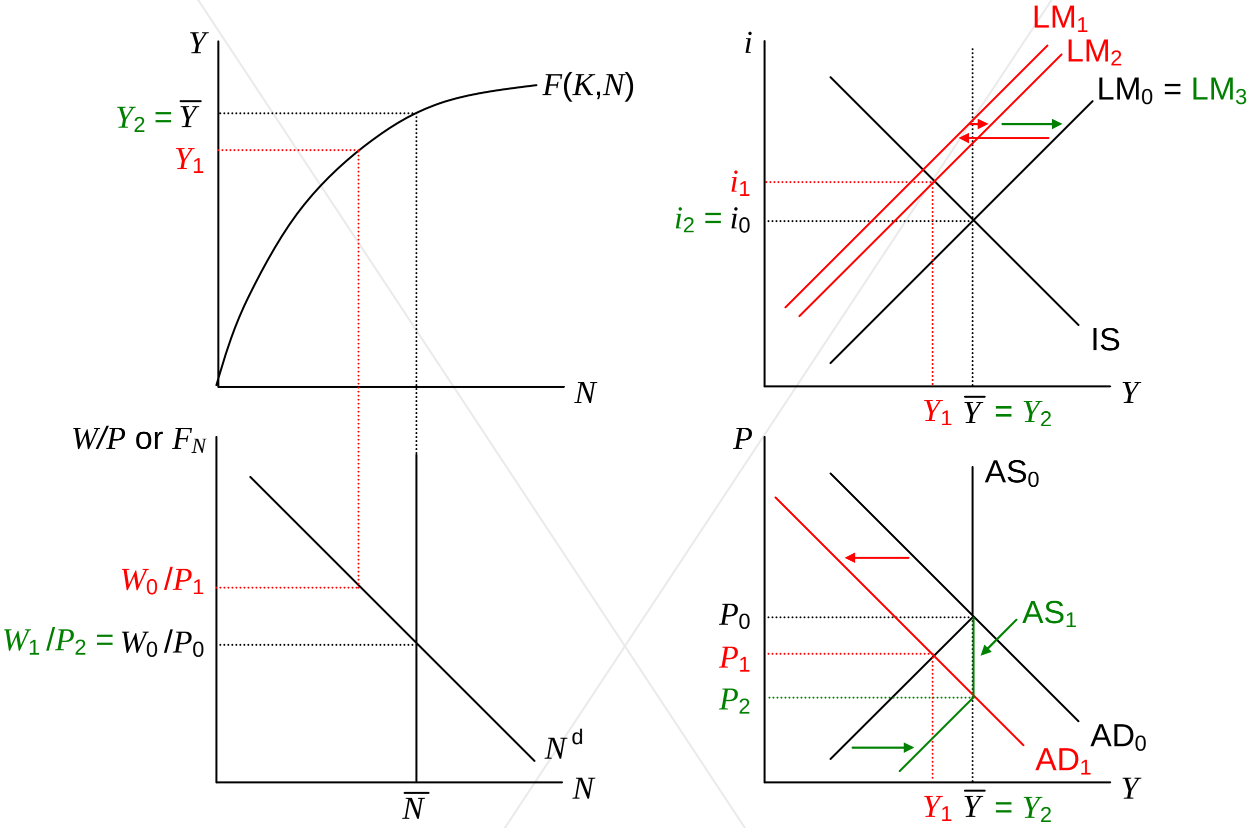
1. Describe the short-run and long-run effects of a decrease in the money supply on (a) the interest rate, (b) prices, (c) output, (d) investment, (e) the exchange rate, and (f) exports.

A decrease in the money supply (M) will cause the LM curve to shift to the left. This in turn causes a decrease in aggregate demand (AD shifts left) resulting in a lower output (Y) and lower price level (P). The lower price level somewhat offsets the decreased aggregate demand by shifting the LM curve slightly back toward it's original position (i.e., right). In the long-run, wages (W) will fall causing aggregate supply (AS) to increase (shift right). This increases output and further lowers the price level, once again shifting the LM curve to the right. Eventually the ratio between wage and price will return to its previous level (although at a lower price) to ensure full employment.

From the graphs, the interest rate (i), P, and Y can be determined directly. Investment (I) follows from the interest rate because $i\uparrow \Rightarrow I\downarrow$ and vice versa ($I' < 0$). The real exchange rate (e) is determined by the function $e(i - i^*)$. Since $e' < 0$, the real exchange rate will also move in the opposite direction as the interest rate (assuming i^* is constant); in this case $i\uparrow$ so $e\downarrow$. This determines the change in exports (EX) because having a falling exchange rate strengthens the dollar, hence lowering exports (i.e., $e\downarrow \Rightarrow EX\downarrow$ and vice versa). In order to find the nominal exchange rate, realize that $e = EP^*/P$. This can be rewritten $E = eP/P^*$. Assuming P^* remains constant, $E\downarrow$ in the short-run because $e\downarrow$ and $P\downarrow$. In the long-run, e is unchanged and $P\downarrow$ so $E\downarrow$.

Summary: $M\downarrow \Rightarrow \text{LM}\downarrow \Rightarrow \text{AD}\downarrow \Rightarrow P\downarrow \Rightarrow \text{LM}\uparrow \Rightarrow W\downarrow \Rightarrow \text{AS}\uparrow \Rightarrow P\downarrow \Rightarrow \text{LM}\uparrow$

	i	P	Y	I	e	E	EX
Short-Run	↑	↓	↓	↓	↓	↓	↓
Long-Run	--	↓	--	--	--	↓	--

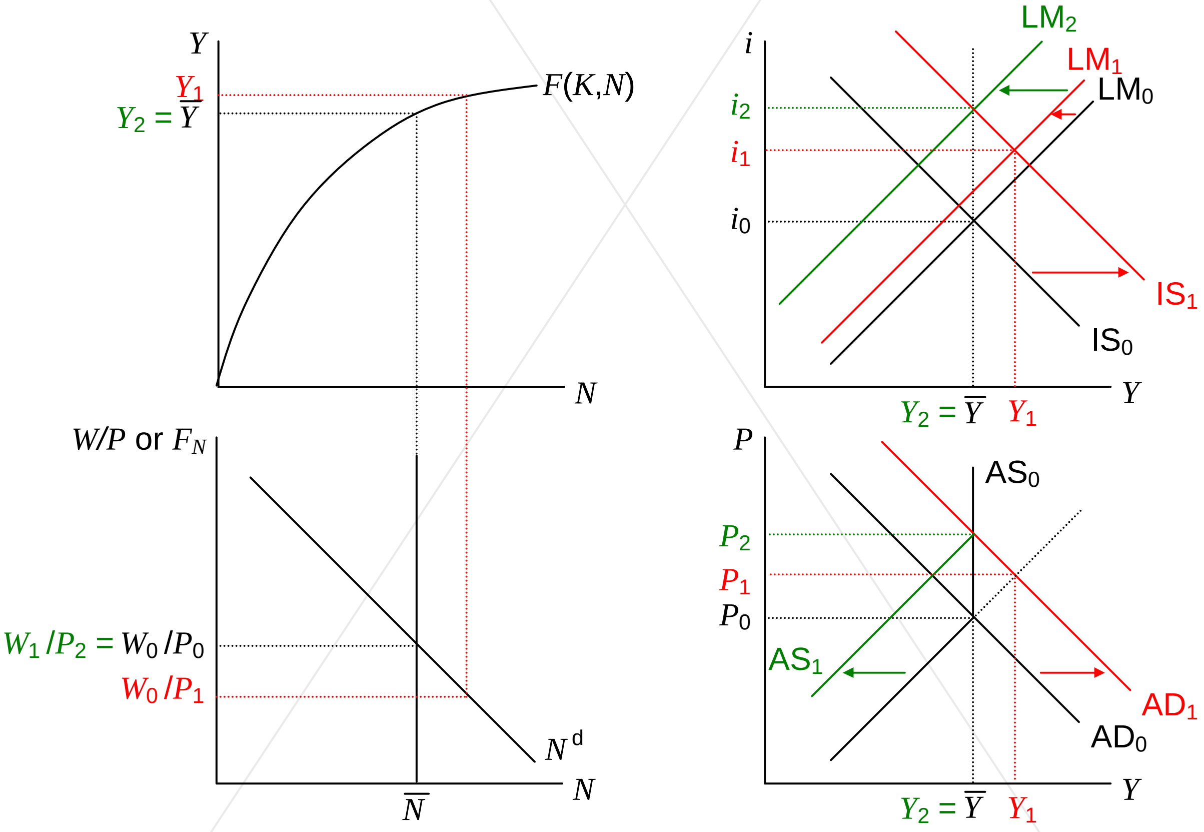


2. Describe the short-run and long-run effects of a decrease in taxes on (a) the interest rate, (b) prices, (c) output, (d) investment, (e) the exchange rate, and (f) exports.

A decrease in taxes (T) effectively increases consumption (C) so it shifts the IS and AD curves to the right resulting in higher P. Output in this case can be viewed as increasing or remaining constant depending on the assumption of the shape of the AS curve. In order to be consistent with the other graphs, we'll assume suppliers can go beyond potential output in the short-run. The higher P causes the LM curve to shift left. In the long-run W will rise resulting in a similar sequence of events described in problem 1 (flip the green arrows). Determining the effects follows the same logic described in the second paragraph of problem 1.

Summary: T↓ ⇒ IS↑ ⇒ AD↑ ⇒ P↑ ⇒ LM↓ ⇒ W↑ ⇒ AS↓ ⇒ P↑ ⇒ LM↓

	i	***P***	***Y***	***I***	***e***	***E***	***EX***
Short-Run	↑	↑	↑	↓	↓	?	↓
Long-Run	↑	↑	--	↓	↓	?	↓

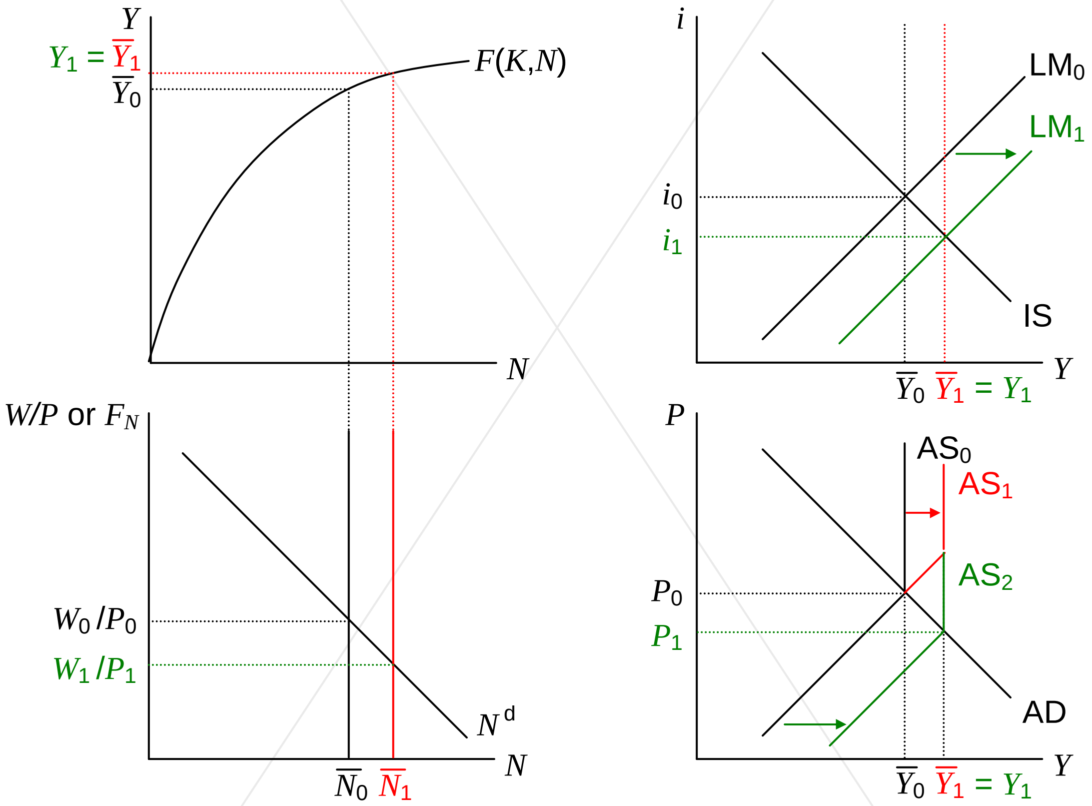


3. Describe the short-run and long-run effects of an increase in the labor force on (a) the interest rate, (b) prices, (c) output, (d) investment, (e) the exchange rate, and (f) exports.

An increase in the labor force ($\overline{N}$) increases potential output ($\overline{Y}$) and shifts the vertical portion AS, but has no other effects in the short-run. In the long-run, W will fall and the same sequence described in problem 1 will occur.

Summary: $\overline{N}$↑ ⇒ W↓ ⇒ AS↑ ⇒ P↓ ⇒ LM↑

	i	P	Y	I	e	E	EX
Short-Run	--	--	--	--	--	--	--
Long-Run	↓	↓	↑	↑	↑	?	↑

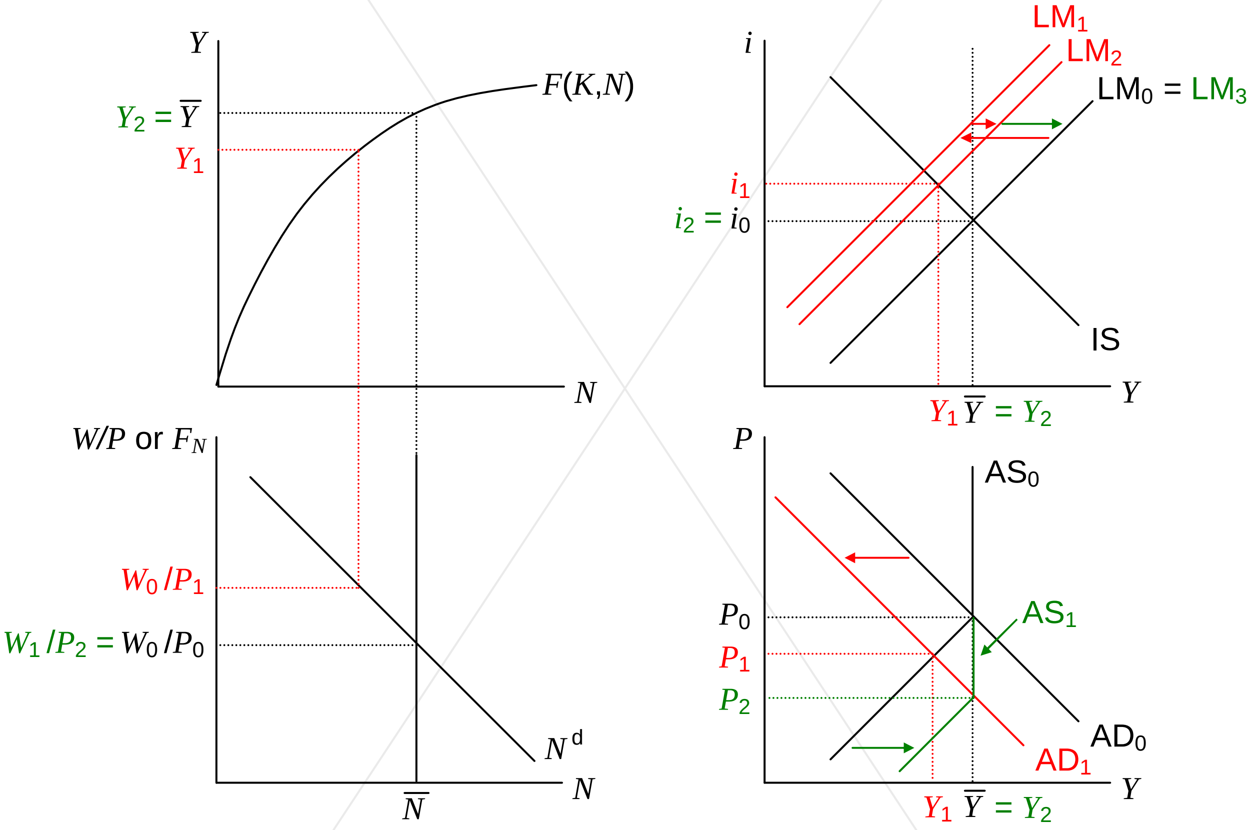


4. Describe the short-run and long-run effects of an increase in the demand for money on (a) the interest rate, (b) prices, (c) output, (d) investment, (e) the exchange rate, and (f) exports.

An increase in the demand for money (L) will cause the LM curve to shift to the left. Everything else will now be the **same as problem 1**.

Summary: L↑ ⇒ LM↓ ⇒ AD↓ ⇒ P↓ ⇒ LM↑ ⇒ W↓ ⇒ AS↑ ⇒ P↓ ⇒ LM↑

	i	P	Y	I	e	E	EX
Short-Run	↑	↓	↓	↓	↓	↓	↓
Long-Run	--	↓	--	--	--	↓	--

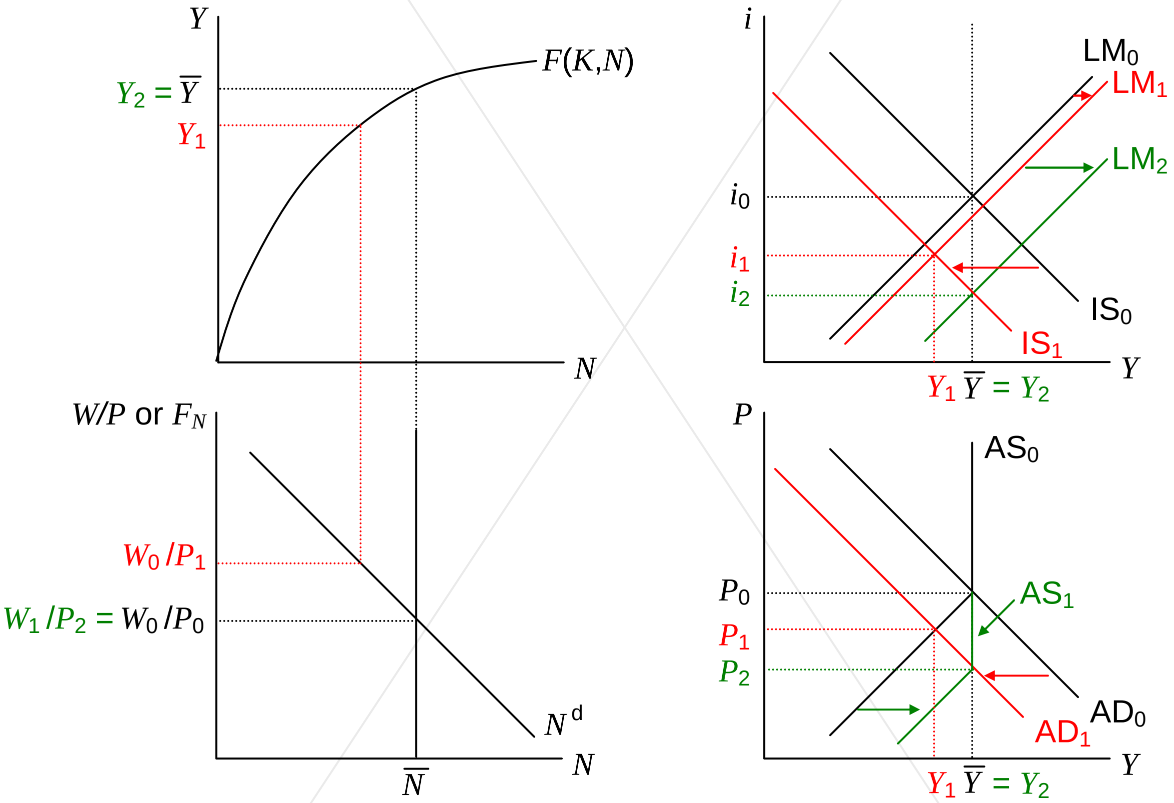


5. Describe the short-run and long-run effects of an increase in the saving rate on (a) the interest rate, (b) prices, (c) output, (d) investment, (e) the exchange rate, and (f) exports.

An increase in the saving rate (*S*) effectively decreases consumption (*C*) so it shifts the IS and AD curves to the left. This mirrors problem 2 (flip the arrows over in the summary). The long long-run will have an increase in output (Solow Model).

Summary: *S*↑ ⇒ IS↓ ⇒ AD↓ ⇒ *P*↓ ⇒ LM↑ ⇒ *W*↓ ⇒ AS↑ ⇒ *P*↓ ⇒ LM↑

	i	*P*	*Y*	*I*	*e*	*E*	*EX*
Short-Run	↓	↓	↓	↑	↑	?	↑
Long-Run	↓	↓	--	↑	↑	?	↑



Documentation

Prof Bomberger helped me in many ways. He confirmed my hunch that problems 1 and 4 were basically the same. He confirmed that the causal flow in the summary for problem 1 was correct. He told me that *Y* could be treated as not changing or increasing depending on the assumption of the shape of AS in problem 2. He confirmed my hunch that there are no short-run effects to increasing the labor supply (problem 3). He discussed how to determine the effects on the exchange rate and exports. Jon Parker and J.C. Zannis both caught errors in problem 4.

Homework 4 Len Cabrera

1. Suppose the time series pattern in yearly income is explained by the following equation:

$$Y_t = 1000 + e_t \text{ where } e_t = u_t + 0.5e_{t-1} \text{ and } u_t \text{ is "white noise".}$$

(a) This is an ARIMA(p,d,q). What are the values for p, d, and q?

$p = 1$	There is 1 lagged e term
$d = 0$	There are no difference terms (i.e., lagged Y)
$q = 0$	There are no lagged u terms

(b) If $e_{2003} = 100$ what is $E_{2003}(Y_{2004})$, $E_{2003}(Y_{2005})$, $E_{2003}(Y_{2031})$?

$E_{2003}(Y_{2004}) = E_{2003}[1000 + u_{2004} + 0.5e_{2003}] = 1000 + 0.5e_{2003} =$
$1000 + 0.5(100) = 1050$
$E_{2003}(Y_{2005}) = E_{2003}[1000 + u_{2005} + 0.5e_{2004}] =$
$E_{2003}[1000 + u_{2005} + 0.5(u_{2004} + 0.5e_{2003})] =$
$1000 + 0.5^2 e_{2003} = 1000 + 0.5^2(100) = 1025$
$E_{2003}(Y_{2031}) = 1000 + 0.5^{28} e_{2003} \approx 1000$

$E_{2003}(Y_{2004}) = 1050$
$E_{2003}(Y_{2005}) = 1025$
$E_{2003}(Y_{2031}) = 1000$

Suppose, instead, that $Y_t = Y_{t-1} + e_t$ where $e_t = u_t + 0.5e_{t-1}$

(c) What are the values for p, d, and q?

$p = 1$	There is 1 lagged e term
$d = 1$	There is 1 difference term (i.e., lagged Y)
$q = 0$	There are no lagged u terms

(d) If $Y_{2003} = 1000$ and $e_{2003} = 100$, what is $E_{2003}(Y_{2004})$?

$E_{2003}(Y_{2004}) = E[Y_{2003} + u_{2004} + 0.5e_{2003}] = Y_{2003} + 0.5e_{2003} =$
$1000 + 0.5(100) = 1050$

$E_{2003}(Y_{2004}) = 1050$

(e) If the average propensity to consume out of permanent income is 0.9 and consumers are "rational", what amount of consumption should occur in 2003?

$C_{2003} = 0.9$(Average expected income)

$$\text{Average expected income} = \frac{1}{(N - 2003 + 1)} \sum_{t=2003}^{N} E_{2003}(Y_t)$$

Permanent Income Theory $\Rightarrow N \rightarrow \infty$
$E_{2003}(Y_{2005}) = E_{2003}[Y_{2004} + u_{2005} + 0.5e_{2004}] =$
$E_{2003}[Y_{2004} + u_{2005} + 0.5(u_{2004} + 0.5e_{2003})] =$
$E_{2003}[Y_{2004}] + 0.5^2 e_{2003} =$

$C_{2003} = 990$

$(Y_{2003} + 0.5e_{2003}) + 0.5^2 e_{2003} =$
$Y_{2003} + (0.5 + 0.5^2)e_{2003}$
$E_{2003}(Y_{2006}) = E_{2003}[Y_{2005} + u_{2006} + 0.5e_{2005}] =$
$E_{2003}[Y_{2005} + u_{2006} + 0.5(u_{2005} + 0.5e_{2004})] =$
$E_{2003}[Y_{2005}] + 0.5^2 E_{2003}[e_{2004}] =$
$(Y_{2003} + (0.5 + 0.5^2)e_{2003}) + (0.5^2 E_{2003}[u_{2004} + 0.5e_{2003}]) =$
$Y_{2003} + (0.5 + 0.5^2 + 0.5^3)e_{2003}$

$$E_{2003}(Y_\infty) = Y_{2003} + \left(\sum_{i=1}^{\infty} 0.5^i\right)e_{2003} = Y_{2003} + e_{2003} = 1000 + 100 = 1100$$

As $N \to \infty$ $C_{2003} = 0.9 \cdot E_{2003}(Y_\infty) = 0.9(1100) = 990$

(f) Under the same assumptions, by how much should consumption change in 2004 if $u_{2004} = 10$?

$\boxed{C_{2003} = 1008}$

$Y_{2004} = Y_{2003} + u_{2004} + 0.5e_{2003} =$
$1000 + 10 + 0.5(100) = 1060$
By similar argument as 1e, we get $E_{2004}(Y_\infty) = Y_{2004} + e_{2004} =$
$1060 + 60 = 1120$
As $N \to \infty$ $C_{2004} = 0.9 \cdot E_{2004}(Y_\infty) = 0.9(1120) = 1008$

2. Suppose that the inflation rate, π, is defined as the change in the log of the price level, P. That is $\pi_t = P_t - P_{t-1}$. Suppose further that inflation is described by the ARIMA process:

$\pi_t = 0.03 + e_t$ where $e_t = u_t + 0.6e_{t-1}$ and u is a "white noise" error term.

(a) If the price level is described as an ARIMA(p,d,q), what are the values for p, d, and q?

$p = 1$ There is 1 lagged e term
$d = 1$ There is 1 difference term (i.e., lagged P)
$q = 0$ There are no lagged u terms

(b) If $P_{2003} = 1.05$ and $P_{2002} = 1.00$, what is the conditional forecast $E_{2003}(P_{2004})$? $E_{2003}(P_{2005})$?

$\boxed{\begin{array}{c} E_{2003}(P_{2004}) = 1.092 \\ E_{2003}(P_{2005}) = 1.1292 \end{array}}$

$P_t = P_{t-1} + 0.03 + u_t + 0.6e_{t-1}$
$P_{2003} = P_{2002} + 0.03 + e_{2003} \Rightarrow$
$e_{2003} = P_{2003} - P_{2002} - 0.03 =$
$1.05 - 1.00 - 0.03 = 0.02$
$E_{2003}(P_{2004}) = E_{2003}[P_{2003} + 0.03 + u_{2004} + 0.6e_{2003}] =$
$P_{2003} + 0.03 + 0.6e_{2003} =$
$1.05 + 0.03 + 0.6(0.02) = 1.092$
$E_{2003}(P_{2005}) = E_{2003}[P_{2004} + 0.03 + u_{2005} + 0.6e_{2004}] =$
$E_{2003}[P_{2004} + 0.03 + u_{2005} + 0.6(u_{2004} + 0.6e_{2003})] =$

$E_{2003}[P_{2004}] + 0.03 + 0.6^2 e_{2003} =$
$(P_{2003} + 0.03 + 0.6e_{2003}) + 0.03 + 0.6^2 e_{2003} =$
$P_{2003} + 2(0.03) + (0.6 + 0.6^2)e_{2003} =$
$1.05 + 2(0.03) + (0.6 + 0.6^2)0.02 = 1.1292$

Documentation.

Prof Bomberger told me consumption equals MPC times the value where income levels off in problems 1e and 1f.

I reviewed my work with Josh Kneifel.